How accurate is the RACH procedure model in LTE and LTE-A?

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***Abstract*— In Long Term Evolution (LTE) networks, User Equipments (UE)s should proceed Random Access CHannel (RACH) procedure to attach to the Base Station and access the channel. One limitation of this procedure is the congestion that may appear when high number of UEs are simultaneously trying to attach to the channel. Such use-case happens when high number of Machine Type Communication (MTC) devices are deployed in one LTE cell. In order to evaluate the RACH performances, in terms of success, collision and idle probabilities, when the traffic load is high, accurate models are needed. However, most of existing models ignore one important constraint, which is the fact that the eNB can knowledge only a limited number of UEs in each RACH round, leading to a mis-formulation of these metrics in the context of LTE, and especially in the presence of high number of devices competing for the channel access.**
In this paper, we tackle the above-mentioned issue by devising a new model for the RACH procedure taking in consideration this constraint. Computer simulation demonstrates that unlike the existing models, our proposed model achieve high accuracy to estimate the performance of the RACH procedure, whatever the traffic load.

***Index Terms*—LTE, LTE-A, RACH procedure, Multichannel slotted ALOHA, balls and bins problem**

I. Introduction

Long Term Evolution (LTE) and LTE-Advanced (LTE-A) come at the forefront of wireless technologies nowadays, due to their characteristics like high peak data rate, flexible bandwidth, the coverage, etc. [1], [2], [3], [4]. LTE represents the access part of Evolved Packet System (EPS), where the another part of EPS is the Evolved Packet Core (EPC). The LTE access part consists of the UEs and the base station, named as evolved Node B (eNB). The main objective of the LTE access network is to provide the UEs the access to the network. The access technology used in LTE and LTE-A is Orthogonal Frequency Division Multiple Access (OFDMA) for downlink and a pre-coded version of OFDMA, namely Single Carrier-FDMA (SC-FDMA), for uplink [5]. On the other hand, the medium access control is based on Slotted ALOHA without reservation, i.e. random access procedure [6]. Many access channels are available for all users, wherein a UE randomly chooses a slot to request the access. One of the advantages of random access schemes is the good performance to deliver short messages [7].

Many works have been done aiming at modeling the RACH procedure, such as [8], [9], [10], [11], [12], [13], [14], [15]. Some of them tried to estimate the number of arrivals at each time, and then to adjust the network's parameters, without introducing a model for the RACH procedure [13], [16]. Others proposed an analytical model for RACH procedure, but either they did not take into consideration the network's constraints, i.e. the number of responses to be sent by the eNB in the RACH procedure is limited [10], [11], or they proposed a proprietary solution, i.e. for certain traffic model [8], [9]. An analytical model is presented in [15], but it does not accurately capture the RACH behavior [17]. In [14], the authors introduced a patch to improve the accuracy of RACH procedure in Network Simulator (NS) 3 without introducing an analytical model. The authors in [18] proposed a method that dynamically allocate resources in the case of Group Paging (GP) in LTE-A networks, where their analysis is principally depended on the ones in [8], [9]. However, none of the proposed models have taken into consideration the fact that eNBs cannot acknowledge more than a certain number of UEs during ne RACH period (noted network constraints in this work) [9], [19], [20]. In this paper, we fill this gap by proposing a new formulation of the performance metrics (i.e., success and collision probabilities), which consider the network constraints.

This paper is organized as follows. Section II presents the problem that arises when the mentioned network constraints are assumed. Some background on the Random Access CHannel (RACH) procedure is presented in Section III. Our proposed model, namely Performance Metrics of RACH procedure with Network Constraints (PMRACH-NC), is inroduced and detailed in Section IV. Performance evaluation of PMRACH-NC is presented in Section V. Finally, conclusions are presented in Section VI.

II. Problem Statement

In order to evaluate the performance of Multi-channel Slotted ALOHA system, used in LTE and LTE-A, the performance metrics, such as success and collision probabilities, should be well defined. Fortunately, theses metrics can be calculated easily when ignoring the network constraints. Using the same reasoning as for the balls and bins problem [21], the above mentioned metrics are obtained as follows:

$$P_S = \binom{M}{1}\left(\frac{1}{R}\right)\left(1-\frac{1}{R}\right)^{M-1} = \frac{M}{R}\left(1-\frac{1}{R}\right)^{M-1}$$

$$P_I = \binom{M}{0}\left(\frac{1}{R}\right)^0\left(1-\frac{1}{R}\right)^{M-0} = \left(1-\frac{1}{R}\right)^M \qquad (1)$$

$$P_C = 1 - P_S - P_I$$

where M is the number of terminals and R is the number of available channels, while $\binom{n}{k}$ is k-combinations. P_S represents the probability that one ball falls in a bin (the success probability in our problem), while P_I is the probability that none of the falls fall in a bin (idle probability). Regarding the collision probability P_C, it is the probability that more than one ball fall in a bin. However, in the RACH procedure after that terminals have randomly chosen a channel and sent the attach request, the network (i.e., eNB) will respond only to the terminals whose signals have been successfully detected and decoded. In addition, the eNB cannot send back responses to all the requests when they are larger than the constraints, i.e. the maximum number of responses N_{ACK} (equation 2). If the number of successful requests is larger than N_{ACK}, the eNB will reply to only N_{ACK} requests, while the remaining ones are considered as collided requests. Because of this constraint, the performance metrics like the success and collision probabilities for the classical problem are biased and no more valid. In Fig. 1, we illustrate this problem,

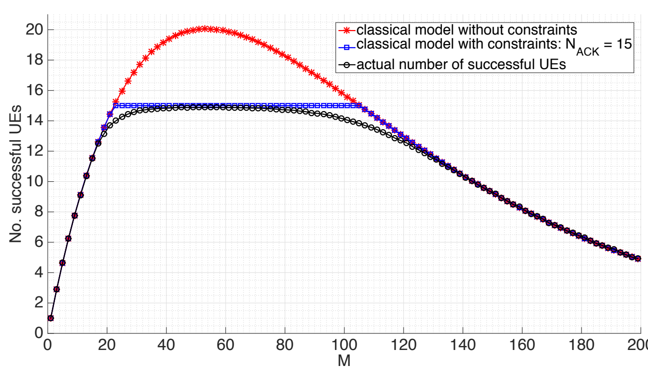

Fig. 1. Number of successful UEs for the classical problem and the actual value; $N_{ACK} = 15$

where the blue curve represents the number of successful UEs when considering the network constraints on the classical form of success probability, i.e. equation (1). We observe that the difference between the classical (blue curve) and the actual values (black curve) is not important. However, this probability is calculated for only one Random Access (RA) slot. When calculating the success probability, or alternatively the number of successful UEs, for a large number of RA slots (as this is the actual case), the observed error will be accumulated until leading to erroneous results. Many studies [8], [22] have been made on the access procedure in LTE and LTE-A, but they did not consider the change of the law of success probability. An interesting model was presented in [8]. However, as stated by the authors the error of the proposed model can reach up to 200% for some configurations, which is completely not acceptable.

III. Background: RACH Procedure in LTE and LTE-A networks

To attach to LTE and LTE-A networks, a UE should first proceed the Random Access CHannel (RACH) procedure. Two forms of RACH procedure exist: contention-based and contention-free RACH procedures. The contention-based procedure is used, in general, when the terminal tries to connect to the network, e.g. to establish a connection or to restore the Uplink synchronization. On the other hand, the contention-free procedure is used when the connection is initiated by the network, e.g. for handover. The RACH procedure consists of the following steps (as illustrated in Fig. 2):

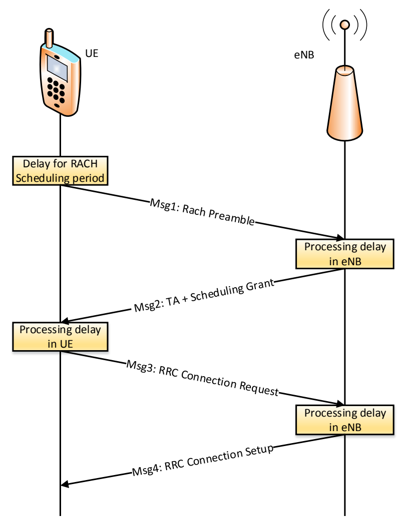

Fig. 2. Random Access CHannel (RACH) procedure

1) *Random Access Preamble Transmission (Msg1):* This step consists of the transmission of a preamble, where a terminal randomly chooses one of the available preambles. As the preamble is randomly chosen, we may have the case where more than one terminal choose the same preamble, and thus causing a collision. In this case, all the terminals having chosen the same preamble will back off and retransmit the preamble later.
2) *Random Access Response Reception (Msg2):* After the preamble transmission, the terminal monitors the Physical Downlink Control CHannel (PDCCH) during certain interval, which is Random Access Response (RAR) window, in order to receive the response message. It should be noted that the maximum number of responses (N_{ACK}) during the RAR window is:

$$N_{ACK} = N_{RAR} \times W_{RAR} \qquad (2)$$

where N_{RAR} is the maximum number of RARs per a response message, and W_{RAR} is the size of the response window in a sub-frame unit. The response message contains many parameters, such as the Timing Advanced (TA) used to adjust the uplink synchronization and the terminal's identifier Temporary Cell-Radio Network Temporary Identifier (TC-RNTI). This message also contains information on the UpLink (UL) resources to be used by the terminal in the next step. When a terminal, with a contention-free procedure, successfully receives the message Msg2, it considers that the RACH procedure has been successfully finished. However, the one with contention-based procedure continues to the next step.

3) *RRC Connection Request (Msg3):* After the reception and processing of the message Msg2, the terminal sends the message Msg3 to request RRC connection from the network.
4) *RRC Connection Setup (Msg4):* This message, sent by the network, is a response message to the precedent one, informing the terminal that the RRC connection has been setup.

IV. Performance metrics of RACH procedure with network constraints (PMRACH-NC)

In this section, we use the balls and bins problem [21] as a basis of our analysis to find the general form of success probability, i.e. the probability that there are many bins/preambles having a certain number of balls/terminals for each. We then mathematically describe the problem and present how it will be solved.

A. General Form of Probability

Let M and R be the numbers of balls/UEs and bins/preambles, respectively. The probability that k_i balls/UEs fall in a bin, let (i), is given by the following equation:

$$\begin{aligned} P_r(\omega_i = k_i) &= \binom{M}{k_i} (\frac{1}{R})^{k_i} (1-\frac{1}{R})^{M-k_i} \\ &= \binom{M}{k_i} (R-1)^{M-k_i} (\frac{1}{R})^{M} \end{aligned} \tag{3}$$

Assuming that the k_i balls have been fall in the bin (i), the probability that another bin, let (j), has k_j balls is given by:

$$\begin{aligned} P_r(\omega_j = k_j/\omega_i = k_i) &= \binom{M-k_i}{k_j} \left(\frac{1}{R-1}\right)^{k_j} \times \\ \left(1-\frac{1}{R-1}\right)^{M-k_i-k_j} &= \binom{M-k_i}{k_j} (R-2)^{M-k_i-k_j} \times \\ &\left(\frac{1}{R-1}\right)^{M-k_i} \end{aligned} \tag{4}$$

Therefore, the probability that there are two bins, let (i) and (j), having k_i and k_j balls, respectively, is:

$$\begin{aligned} &P_r(\omega_i = k_i, \omega_j = k_j) = P_r(\omega_i = k_i) P_r(\omega_j = k_j/\omega_i = k_i) \\ &= \frac{\binom{M}{k_i}(R-1)^{M-k_i}}{R^M} \frac{\binom{M-k_i}{k_j}(R-2)^{M-k_i-k_j}}{(R-1)^{M-k_i}} \\ &= \binom{M}{k_i}\binom{M-k_i}{k_j} (R-2)^{M-k_i-k_j}/R^M \\ &= \binom{M}{k_i}\binom{M-k_i}{k_j} \left(\frac{1}{R}\right)^{k_i+k_j} \left(1-\frac{2}{R}\right)^{M-k_i-k_j} \end{aligned} \tag{5}$$

By induction, we can find that the probability that r bins/preambles having k_i ; $i = 1:r$ balls/UEs for each is equal to:

$$\begin{aligned} P_r(\omega_{1:r} = k_{1:r}) &= \binom{M}{k_1}\binom{M-k_1}{k_2} \ldots \binom{M-\sum_{i=1}^{r-1} k_i}{k_r} \times \\ &\left(\frac{1}{R}\right)^{\sum_{i=1}^{r} k_i} \left(1-\frac{r}{R}\right)^{M-\sum_{i=1}^{r} k_i} \end{aligned} \tag{6}$$

This equation will be later used to formulate and demonstrate the new law of success probability.

B. Problem Formulation

Generally, the success probability is given by the following equation:

$$P_s = P_r(\omega_i = 1) = \binom{M}{1}\left(\frac{1}{R}\right)\left(1-\frac{1}{R}\right)^{M-1} \simeq \frac{M}{R} e^{-\frac{M}{R}} \tag{7}$$

However, the eNB cannot reply to more than N_{ACK} UEs, even if the number of successful ones exceeds this value. Therefore, a reformulation of the success probability (equation 7) should be done to take into consideration this constraint.

Let N_ω be the number of successful preambles, i.e. there are only N_ω preambles (and thus N_ω UEs) that are successful whereas the others are not. The success probability P_s can be decomposed as follows:

$$\begin{aligned} &P_s = P_r(N_\omega = 1 \cup N_\omega = 2 \cup \ldots \\ &\ldots \cup N_\omega = R-1 \cup N_\omega = R) = \sum_{r=1}^{R} P_r(N_\omega = r) \end{aligned} \tag{8}$$

where $P_r(N_\omega = r)$, or alternatively $P_r(\omega_{1:r} = 1, \omega_{r+1:R} \neq 1)$, is the probability that there are only r successful UEs. Note that the intersection of N_ω in the precedent equation is zero. When the network's constraints are applied, the probability $P_r(N_\omega = r)$ will be affected only when $r > N_{ACK}$. Therefore, the success probability becomes:

$$\begin{aligned} P_s &= \sum_{r=1}^{N_{ACK}} \frac{r}{r} P_r(N_\omega = r) + \sum_{r=N_{ACK}+1}^{R} \frac{N_{ACK}}{r} P_r(N_\omega = r) \\ &= \sum_{r=1}^{N_{ACK}} P_r(N_\omega = r) + \sum_{r=N_{ACK}+1}^{R} \frac{N_{ACK}}{r} P_r(N_\omega = r) \end{aligned} \tag{9}$$

To better understand the precedent equation, we will rely on the following logical argument. Imagine that there is a large number of experiences, the success probability can be approximated by the following equation;

$$P_s = \lim_{n \longrightarrow \infty} \frac{\sum_{i=1}^{n} N_{s_i}}{n} \tag{10}$$

where N_{s_i} is the total number of successful UEs in the experiment (i), and n is the total number of experiments. When the network's constraints are not applied, the success probability will be easily calculated by counting the number of successful UEs at each experiment and then dividing them by the total number of experiments, i.e. we will take all the successful UEs. However, when the network's constraints are applied, i.e. the network sends back at most N_{ACK} responses, some changes should be made. In this case, for each experiment we count the number of successful UEs. If they are smaller than or equal to N_{ACK}, we then take all of them. Otherwise, we will take just N_{ACK} successful ones. That is, for the experiments in which the number of successful UEs is larger than N_{ACK}, we take only N_{ACK} successful ones out of the total r successful ones. This change can be formulated mathematically by the equation 9. However, in order to calculate the success probability given by the equation 9, we need to calculate the probability that there is only a certain number of successful UEs, i.e. $P_r(N_\omega = r)$. Due to the space's limitation, we introduce here only the main steps, where a comprehensive demonstration can be found in [23]. In order to calculate the new law of the success probability, the first bin/preamble is considered. Let's begin by the probability that there are only $(R-2)$ successful UEs/balls, i.e. $(N_\omega = R-2)$. The probability in this case can be given by the following equation:

$$P_r(\omega_{1:R-2} = 1, \omega_{R-1:R} \neq 1) = \binom{R-1}{(R-2)-1} \times P_r(\omega_{1:R-2} = 1) P_r\left(\omega_{R-1} \neq 1, \omega_R \neq 1\right) \quad (11)$$

As we consider the first bin/preamble, the rest $(R-2)-1$ successful UEs can come at any order in the rest $(R-1)$ bins/preambles, i.e. there are $(R-2)-1$ combinations in $(R-1)$ bins/preambles. The term $P_r\left(\omega_{R-1} \neq 1, \omega_R \neq 1\right)$ can be formulated as follows:

$$P_r\left(\omega_{R-1} \neq 1, \omega_R \neq 1\right) = \sum_{i=0}^{M-R+2} P_r(\omega_{R-1} = i, \omega_R = M-R+2-i) - P_r(\omega_{R-1} = 1, \omega_R = M-R+1) - P_r(\omega_{R-1} = M-R+1, \omega_R = 1) \quad (12)$$

which is equal to all the potential possibilities minus the ones in which a bin/preamble is successful. Applying the general form of probability (equation 6) on the equations 11 and 12, the probability that there are only $(R-2)$ successful balls/UEs becomes:

$$P_r(\omega_{1:R-2} = 1, \omega_{R-1:R} \neq 1) = \binom{R-1}{(R-2)-1} \binom{M}{1} \ldots \binom{M-R+3}{1} \left(2^{M-R+2} - 2\binom{M-R+2}{1}\right) \left(\frac{1}{R}\right)^M \quad (13)$$

By using the same methodology, we can find that the probabilities to have only $(R-3)$ and $(R-4)$ successful balls/UEs are equal to [23]:

$$P_r(\omega_{1:R-3} = 1, \omega_{R-2:R} \neq 1) = \binom{R-1}{(R-3)-1} \times P_r(\omega_{1:R-3} = 1)\left(I_1 - I_2 - I_3 - I_4 + I_5 + I_6 + I_7\right) = \binom{R-1}{(R-3)-1} \binom{M}{1} \binom{M-1}{1} \ldots \binom{M-R+4}{1} \times \left(3^{M-R+3} - 3\binom{M-R+3}{1} \times 2^{M-R+2} + 3\binom{M-R+3}{1}\binom{M-R+2}{1}\right) \left(\frac{1}{R}\right)^M \quad (14)$$

$$P_r\left(\omega_{1:R-4} = 1, \omega_{R-3:R} \neq 1\right) = \binom{R-1}{(R-4)-1} \binom{M}{1} \ldots \times \binom{M-R+5}{1} \{I_1 - I_2 + I_3 - I_4\} = \binom{R-1}{(R-4)-1} \binom{M}{1} \ldots \times \binom{M-R+5}{1} \left\{4^{M-R+4} - \binom{4}{1}\binom{M-R+4}{1} 3^{M-R+3} + \binom{4}{2}\binom{M-R+4}{1}\binom{M-R+3}{1} 2^{M-R+2} - \binom{4}{3} \times \binom{M-R+4}{1}\binom{M-R+3}{1}\binom{M-R+2}{1}\right\} \left(\frac{1}{R}\right)^M \quad (15)$$

By induction from equations 13, 14, and 15, and doing some formulation, we find that the probability that there are only (r) successful balls/UEs is equal to:

$$P_r(N_\omega = r) = \binom{R-1}{r-1} M! \sum_{\alpha=0}^{R-r} (-1)^\alpha \binom{R-r}{\alpha} \times \frac{(R-r-\alpha)^{M-r-\alpha}}{(M-r-\alpha)!} \left(\frac{1}{R}\right)^M \quad (16)$$

After calculating the probability $P_r(N_\omega = r)$, we can easily calculate the new success probability by equation 9. It should be noted that this change will affect the success and collision probabilities, while the idle probability remains unchanged.

V. Performance Evaluation

In order to validate our proposed analytical model PMRACH-NC and compare it accuracy to the Classical Model (CM), computer simulations were carried out by using a custom C++ simulator (discrete-event simulator for the RACH procedure). For this purpose, the simulations will be done on two parts. In the first part, we consider that there are $(R = 54)$ preambles and the number of UEs varies from (1) to (200). The simulations were developed based on Monte-Carlo approach. As a criterion of evaluation, we consider the Relative Error (RE), which represents the gap between the analytical model and simulation results, i.e. the analytical model accuracy. It is given by the following equation:

$$RE = \frac{sim - ana}{sim} \times 100 \quad (17)$$

where ana and sim are the results obtained by the analytical model and simulation, respectively. Regarding the second part of simulation, we consider that UEs are activated using a

TABLE I
BASIC SIMULATION PARAMETERS

Notations	Definition	Values
R	No. preambles in a random access slot	54
BI	Backoff indicator in a sub-frame unit	20
$N_{PT_{max}}$	Maximum number of preamble transmission	10
N_{RAR}	Max. No. RARs that can be carried in one response message	3
W_{RAR}	The size of the random access response window in a sub-frame unit	5
N_{ACK}	Max. No. MTCs that can be acknowledged within the RAR window	$N_{ACK} = N_{RAR} \times W_{RAR}$
$PRACH_{config\_indx}$	PRACH configuration index	$PRACH_{confic\_indx} = 6$
T_{RA_REP}	The interval between two consecutive Random Access (RA) slots	5

uniform distribution (e.g., group paging case [19]) during an interval equal to:

$$I = 1 + (N_{PT_{max}} - 1)\left\lceil \frac{T_{RAR} + W_{RAR} + W_{BO}}{T_{RA\_REP}} \right\rceil \tag{18}$$

where $N_{PT_{max}}, W_{BO}$, and T_{RA_REP} are the maximum number of preamble transmission, the size of backoff window, and the interval between two consecutive RA slots, respectively. On average, there are N_{ACK} new arrivals at each RA slot. Furthermore, the parameters of RACH procedure specified by Table 6.2.2.1 in [20], and also the control-plane latency analysis specified in Table $B.1.1.1-1$ in [24] are used in the simulations. For the sake of simplicity and to better understand the problem, the probability to detect a preamble by eNB is set to be one. The main parameters are summarized in Table I.

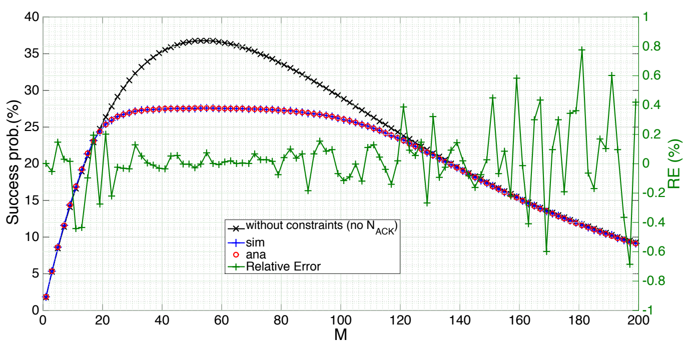

Fig. 3. Success probability for simulation and analytical model PMRACH-NC: $N_{ACK} = 15$

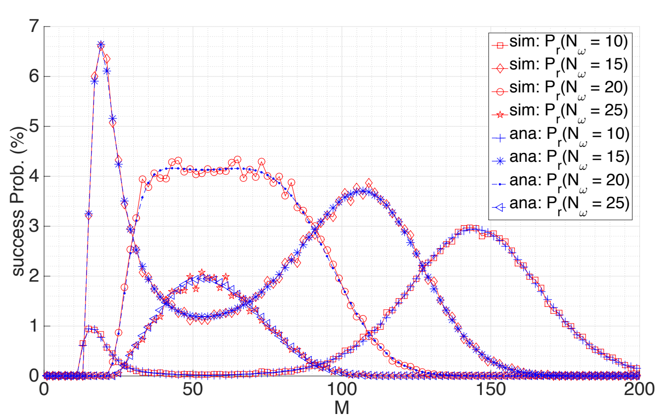

Fig. 4. The probability of existing only a certain number of successful devices: $N_{ACK} = 15$

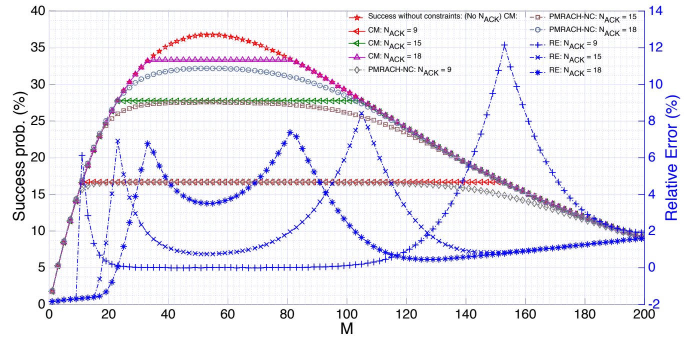

Fig. 5. Success probability and the corresponding relative error for different values of N_{ACK}

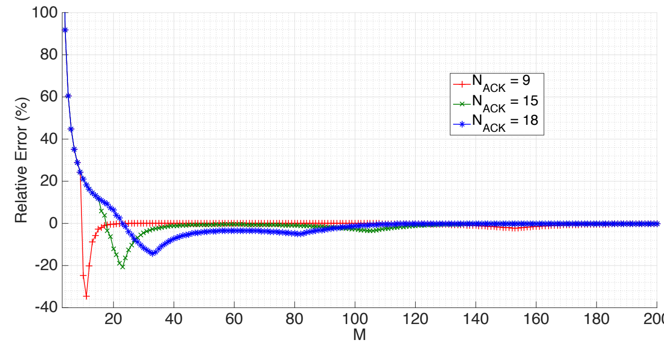

Fig. 6. Relative error of collision probability for different values of N_{ACK}

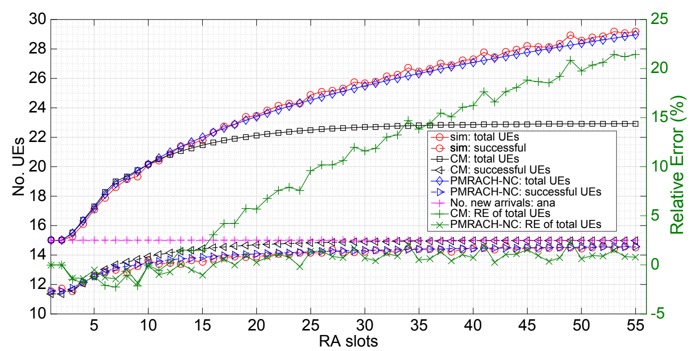

Fig. 7. Number of successful and total UEs at each RA slot for the considered methods: $N_{ACK} = 15$

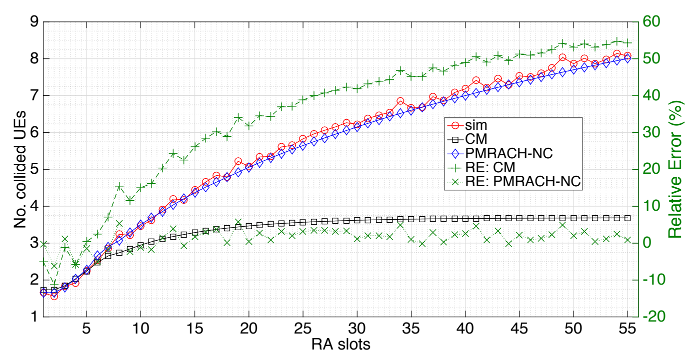

Fig. 8. Number of collided preambles at each RA slot for the considered methods: $N_{ACK} = 15$

Fig. 3 illustrates the success probability obtained by the classical model CM, simulation, and the proposed analytical model PMRACH-NC. This figure confirms the validity of PMRACH-NC, since the simulation and the analytical curves are mostly identical. This observation is further confirmed by the relative error, which is less than 1%. It is worth noting that this error tends to zero when the number of experiments tends to the infinity. Results in Fig. 4 show the probability of existing only a certain number of successful devices ($N_\omega = 10 : 5 : 25$). The same trend is observed in this figure, i.e. the high accuracy of PMRACH-NC.

Figs. 5 and 6 show the difference between CM and PMRACH-NC for different values of N_{ACK}. We clearly see that the relative error, defined as $100 \times (classical\_model - new\_model)/new\_model$, for the success probability depends on N_{ACK} and it can exceed 10%. Further, this error is even worse for the collision probability, where it can reach more than 40%, confirming the inaccuracy of the classical model.

Figs. 7 and 8 illustrate the results obtained in the second part of the simulation, i.e. UEs activated according to Uniform distribution. From Fig. 7, we clearly see the gap between CM and the simulation results, demonstrating the issue raised in section II. Unlike CM, PMRACH-NC is highly accurate in estimating the success probability (or alternatively the number of successful UEs), the collision probability, and the total number of UEs at each RA slot. The accuracy of PMRACH-NC is confirmed by the relative errors of the total number of UEs (Fig. 7) and the number of collided UEs (Fig. 8), which are merely equal to zero, while these errors are more than 20% for the total number of UEs and more than 50% for the collided UEs for the classical model CM.

VI. Conclusion

The RACH procedure represents the key point of LTE performances, since most of the radio resources are negotiated at the UEs attach procedure. Accordingly, it is a vital need to have a good analytical model that better represents this procedure. Indeed, such accurate model will help to better evaluate the network's performance and also the methods (algorithms) tackling with the overload and congestion in the Radio Access Network (RAN) part of the network. In this paper, after showing the inefficiency of the current models, we introduced a new model of success probability that better matches the RACH procedure. Simulation results have demonstrated not only the accuracy of the new model of success probability, but also the inaccuracy of the classical one. The inefficiency of classical model is confirmed by the relative error that can exceed 20% and 50% on the total number of UEs and the collision probability, respectively.

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