A Novel Fuzzy Concept-Cognitive Learning Model with Attribute Fluctuation and Concept Clustering

Xianwei Xin, Haoyuan Yu, Zhanao Xue, Chenyang Wang*, *Member, IEEE*, Tarik Taleb, *Senior Member, IEEE*

***Abstract*—Concept-cognitive learning (CCL) is a paradigm that simulates human concept learning by processing given cues through specific cognitive models. However, existing CCL models face significant limitations, such as weak correlations between attributes and decisions, high redundancy within the concept space, and suboptimal learning performance. To address these issues, this paper introduces an Attribute Fluctuation-Based CCL (AFFCCL) model. First, a novel measurement method for attribute fluctuation is proposed, based on the variation range of attribute membership degrees. To mitigate redundancy in the concept space, a fuzzy granular concept space is constructed using the concept contribution degree. Second, the model leverages the semantic richness of concepts by integrating similar fuzzy granular concepts, thereby constructing a clustering space. From this, upper and lower approximation spaces are derived. Finally, extensive experiments conducted on multiple benchmarkdatasets demonstrate that the proposed AFFCCL model outperforms representative fuzzy CCL models, neural-network-basedclassifiers, and traditional similarity-based approaches in termsof accuracy, interpretability, and robustness.**

***Index Terms*—Concept-cognitive learning (CCL), Granular computing, Attribute fluctuation, Concept clustering.**

I. Introduction

COGNITIVE computing, a self-learning paradigm within artificial intelligence, leverages advanced algorithms and techniques to uncover valuable knowledge and its underlying structural relationships from vast amounts of data [1]. By imbuing computer systems with cognitive capabilities such as comprehension, learning, reasoning, and natural language processing, cognitive computing enables the effective handling of complex decision-making tasks [2]. Among the key methodologies driving cognitive computing, knowledge mining based on formal concept analysis (FCA) [3] has emerged as a powerful tool. FCA, grounded in set theory and lattice theory, reveals hidden structural patterns and relationships within data by analyzing associations between objects and their attributes. Despite its advantages, FCA is hindered by challenges such as high construction costs and concept redundancy and low dynamic adaptability, which limit its broader application and development.

In recent years, the advancement of concept-cognitive learning (CCL) has introduced a novel impetus to cognitive computing. CCL, an interdisciplinary field that integrates concepts from the structured representation of FCA, the upper and lower approximation of rough sets to deal with uncertainty [4], the multi-granularity computation of granular computing [5], and cognitive computing, enables the extraction of valuable knowledge from given cues using specific cognitive models. It addresses several limitations of traditional FCA, particularly the high construction costs associated with concept lattices. This has led to increasing scholarly attention on CCL. Wu et al. [6] established a close relationship between multi-label concepts and features using degree information, effectively capturing their complex interactions. Yang et al. [7] employed the Jaccard similarity coefficient to propose an attribute and object-based CCL method, enhancing the diversity of learning results by exploiting marginal concepts. To overcome the issue that the existing CCL models make insufficient use of the skill context and negative information, Xu et al. [8] proposed a transformation method to transform between different three-way information granules, which makes full use of context information. Furthermore, Xu et al. [9] advanced the field by integrating positive and negative labels to achieve more accurate knowledge representation.

In real-world scenarios, the data often exhibit fuzziness. To address this uncertainty, fuzzy set theory [10] has been extensively applied across various domains. In feature selection [11], a more flexible feature description is provided by quantifying the degree of membership of features to categories. When measuring uncertainty [12], the membership function is used to describe the ambiguity of data directly, so as to describe uncertainty more accurately and provide a variety of uncertainty measurement methods. In neural networks [13], fuzzy neural networks are constructed to improve the generalization ability and robustness of neural networks. Fuzzy set theory strengthens the ability of cognitive computing to imitate human reasoning by simulating the progressiveness, fuzziness and situational adaptability in the process of human

This work is supported by the National Natural Science Foundation of China (Grant No. 62406104), the Key Research Project for Higher Education Institutions in Henan Province (Grants No. 24A520019 and No. 25A520029), the Science and Technology Research Project of Henan Province (Grants No. 252102210056 and No. 252102211013) and in part by the National Social Science Foundation of Social Science of China (Grant No. 23STA031). (Corresponding author: Chenyang Wang.)

Xianwei Xin is with the School of Henan Normal University, Computer and Information Engineering, Xinxiang, 453007, Henan, China, and with the Key Laboratory of Computational Intelligence and Chinese Information Processing of Ministry of Education, 237016, Taiyuan, China (e-mail: xinxianwei@mail.bnu.edu.cn).

Haoyuan Yu is with the School of Henan Normal University, Computer and Information Engineering, Xinxiang, 453007, Henan, China (e-mail: yuhaoyuan@stu.htu.edu.cn).

Zhanao Xue is with the Key Laboratory of Artificial Intelligence and Personalized Learning in Education of Henan Province. Xinxiang, 453007, Henan, China (e-mail: xuezhanao@163.com).

Chenyang Wang is with the School of College of Shenzhen University, Computer and Software Engineering, Shenzhen, 518060, Guangdong, China (e-mail: chenyangwang@ieee.org).

Tarik Taleb is with the Faculty of Electrical Engineering and Information Technology, Ruhr University Bochum, 44801 Bochum, Germany (e-mail: tarik.taleb@rub.de).

semantic generalization, enabling cognitive models to process fuzzy data in the real world. Its integration with CCL has attracted significant attention. Deng et al. [14] leveraged fuzzy granular concepts to define construction operators, proposing an incremental fuzzy granular CCL model. Aiming at the problem that multi-view data is difficult to be effectively represented and fused by existing single-view fuzzy concept cognitive learning models, Wang et al. [15] proposed a multi-view fuzzy CCL model, which reconstructs the fuzzy attributes of each view, effectively solving the knowledge representation and fusion problem of multi-view data, and realizing the effective resolution of representational inconsistency across parallel perspectives. In order to overcome cognitive barriers and emulate imaginative processes observed in human brains, Ding et al. [16] designed an interval-intent fuzzy concept re-cognition learning model, which effectively simulated the cognitive process akin to that of human phenomena. Guo et al. [17], [18] introduced three-way decision models and proposed a fuzzy-granular three-way CCL model to improve the dynamic updating mechanism of the incremental three-way CCL model under fuzzy context, so as to address the imbalance between high-dimensional data and knowledge evolution limitations. Although these contributions have propelled the CCL theory forward, current models still face several key challenges:

1) **Weak Attribute-Decision Correlation.** Existing models often overlook the relationship between attributes and decisions and lack a dynamic weighting mechanism that can reflect the varying importance of attributes and concepts across different decision classes.
2) **Limited Concept Correlation.** Current fuzzy CCL models primarily rely on the extent of concepts to compute correlations, failing to fully exploit the semantic intent information of concepts to achieve higher classification performance.
3) **Suboptimal Classification.** Many CCL models rely solely on similarity or structural information between concepts for sample prediction, thereby underutilizing the full range of concept information, which limits their classification effectiveness.

To address these issues, this paper proposes an innovative approach within fuzzy formal contexts. Specifically, it introduces a method to measure attribute fluctuation by analyzing the amplitude of membership degree variations across decision classes. This approach reduces redundancy in the concept space through the degree of concept contribution, generating a γ fuzzy granular concept space that accommodates varying decision classes. Furthermore, the paper introduces a novel calculation for concept correlation by incorporating both the extent and intent of concepts, facilitating the generation of pseudo-concepts with enhanced generalization capabilities via concept clustering. Inspired by rough set theory's upper and lower approximations, we propose a new label prediction mechanism that integrates structural and similarity information between concepts. Finally, extensive experiments are conducted, comparing the proposed model's performance with 20 classification algorithms across 15 datasets, demonstrating its superior classification ability.

The primary contributions of this work are as follows:

1) **Dynamic Attribute Utilization.** By measuring attribute fluctuation based on membership degree variations, we effectively capture the differential emphasis of attributes across decision classes, enhancing model dynamic adaptability.
2) **Integration of Intent Information.** The inclusion of concept intent information in the correlation measurement reduces the concept space, thereby simplifying the model and improving efficiency.
3) **Enhanced Label Prediction.** A novel label prediction approach, which combines similarity and structural information, significantly increases the model classification performance.

The remainder of this paper is organized as follows. Section II provides a brief overview of key concepts in the CCL model. Section III discusses the construction of the γ-fuzzy granular concept space. Section IV introduces the concept clustering and label prediction methodology based on rough set approximations. Section V presents an experimental validation of the effectiveness of the model. Finally, Section VII concludes the paper and outlines directions for future research.

II. PRELIMINARIES

A. Classical Formal Context and Concept

Definition 2.1: [3] Let the triple (U, A, I) be a classical formal context, where $U = \{x_1, x_2, \ldots, x_n\}$ represents a non-empty set of objects, with each $x_i (i \leq n)$ denoting a specific object; $A = \{a_1, a_2, \ldots, a_m\}$ is a non-empty set of attributes, where each $a_j (j \leq m)$ corresponds to a single attribute; and $I \subseteq U \times A$ is a binary relationship between U and A. For any $X \subseteq U$ and $B \subseteq A$, let $\mathfrak{L}(X)$ denote the set of attributes shared by all objects in X, and $\mathfrak{H}(B)$ denote the set of objects that possess all the attributes in B.

Definition 2.2: [19] Let (U, A, I) be a classical formal context, for any $X \subseteq U$ and $B \subseteq A$, if $\mathfrak{L}(X) = B$ and $\mathfrak{H}(B) = X$, then the pair (X,B) is called a formal concept (or simply a concept), where X represents the extent and B represents the intent of the concept (X,B).

Definition 2.3: [3] Let (U, A, I) be a classical formal context, for an object $x \in U$, the pair $(\mathfrak{H}\mathfrak{L}(x), \mathfrak{L}(x))$ is called the object granular concept. Similarly, for an attribute $a \in A$, the pair $(\mathfrak{H}(a), \mathfrak{L}\mathfrak{H}(a))$ is called the attribute granular concept.

Theorem 2.1: [3] Let (U, A, I) be a classical formal context. For any concept (X,B), the following properties holds:

$$(X, B) = \bigvee_{x \in X} (\mathfrak{H}\mathfrak{L}(x), \mathfrak{L}(x)) = \bigwedge_{a \in A} (\mathfrak{H}(a), \mathfrak{L}\mathfrak{H}(a)). \quad (1)$$

This theorem expresses that the formal concept (X,B) can be represented both as the least upper bound of the object granular concepts and as the greatest lower bound of the attribute granular concepts.

Definition 2.4: [6] Let (U, A, I) and (U, D, J) be two classical formal contexts. Similar to the operators $\mathfrak{L}$ and $\mathfrak{H}$, the operators $\mathfrak{L}^D : 2^U \to 2^D$ and $\mathfrak{H}^D : 2^D \to 2^U$ are defined. For any $d_1, d_2 \in D$, if $\mathfrak{H}^D(d_1) \bigcap \mathfrak{H}^D(d_2) = \emptyset$, then the quintuple (U, A, I, D, J) is called a classical formal decision context.

B. Fuzzy Formal Context and Concept

Definition 2.5: [20] Let the triple $(U, A, \tilde{I})$ be a fuzzy formal context, where $U = \{x_1, x_2, \ldots, x_n\}$ represents a non-empty set of objects, with each $x_i(i \leq n)$ denoting a specific object; $A = \{a_1, a_2, \ldots, a_m\}$ is a non-empty set of attributes, where each $a_j(j \leq m)$ corresponds to a specific attribute; $\tilde{I} : U \times A \rightarrow [0, 1]$. For each $x \in U$ and $a \in A$, $\mu(x, a) \in [0, 1]$, where $\mu(x, a)$ denotes the degree of membership of x to attribute a in the fuzzy formal context.

Let L^A represent the set of all fuzzy sets on A. For any $X \subseteq 2^U$ and $\tilde{B} \subseteq L^A$, the following operators $\tilde{\mathfrak{L}} : 2^U \rightarrow L^A$ and $\tilde{\mathfrak{H}} : L^A \rightarrow 2^U$ are defined:

$$\tilde{\mathfrak{L}}(X)(a) = \bigwedge_{x \in X} \tilde{I}(x, a), a \in A,$$

$$\tilde{\mathfrak{H}}(\tilde{B}) = \{x \in X : \forall a \in A, \tilde{I}(x, a) \geq \tilde{B}(a)\}.$$

If $\tilde{\mathfrak{L}}(X)=\tilde{B}$ and $\tilde{\mathfrak{H}}(\tilde{B})=X$, then the pair $(X, \tilde{B})$ is called a fuzzy concept, where X is the extent and $\tilde{B}$ is the intent of the fuzzy concept $(X,\tilde{B})$. For each $x \in U$, the pair $(\tilde{\mathfrak{H}}\tilde{\mathfrak{L}}(x),\tilde{\mathfrak{L}}(x))$ is called a fuzzy object granular concept.

For two fuzzy concepts $(X_1,\tilde{B}_1), (X_2,\tilde{B}_2) \in L(U, A, \tilde{I})$, a partial order relationship is defined as $(X_1,\tilde{B}_1) \leq (X_2,\tilde{B}_2)$, which holds if and only if $X_1 \subseteq X_2$ or $\tilde{B}_2 \subseteq \tilde{B}_1$. In this case, $(X_1,\tilde{B}_1)$ is called a sub-concept of $(X_2,\tilde{B}_2)$, and $(X_2,\tilde{B}_2)$ is called a parent concept of $(X_1,\tilde{B}_1)$.

Definition 2.6: [20] Let $(U, A, \tilde{I})$ be a fuzzy formal context, and (U, D, J) be a classical formal context. The quintuple $(U, A, \tilde{I}, D, J)$ is called a fuzzy formal decision context, where $D = \{d_1, d_2, \ldots, d_l\}$ is a decision relation. Additionally, $\tilde{J} : U \times D \rightarrow \{0, 1\}$, $A \bigcap D = \emptyset$.

For each $x \in U$, there exists a unique $\exists! d \in D$ that $J(x, d) = 1$, and thus, U can be partitioned into $U/D = \{c_1, c_2, \ldots, c_l\}$, where $c_k(k \leq l)$ represents the set of objects under decision attribute k.

Definition 2.7: [21] Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context. The fuzzy object granular concept space $\epsilon\varepsilon_k(k \leq l)$, corresponding to the decision class $c_k(k \leq l)$ is expressed as:

$$\epsilon\varepsilon_k = \{(\tilde{\mathfrak{H}}\tilde{\mathfrak{L}}(x),\tilde{\mathfrak{L}}(x)) | x \in c_k\}.$$

Definition 2.8: [14] Let $(U, A, \tilde{I})$ be a fuzzy formal context. For $x \in U$, $a \in A$, the operator $*$ is defined as follows:

$$(x, a)^* = \{y \in U | \tilde{I}(y, a) > \tilde{I}(x, a)\}, \tag{2}$$

where $(x, a)^*$ represents the set of objects whose membership values under attribute a exceed those of $\tilde{I}(x, a)$. According to Definition 2.5, $(\tilde{\mathfrak{H}}\tilde{\mathfrak{L}}((x, a)^*),\tilde{\mathfrak{L}}((x, a)^*))$ is a fuzzy concept.

Theorem 2.2: [14] Let $(U, A, \tilde{I})$ be a fuzzy formal context. Then, for any $x \in U$ and $a \in A$, the following equality holds:

$$(\tilde{\mathfrak{H}}\tilde{\mathfrak{L}}((x, a)^*),\tilde{\mathfrak{L}}((x, a)^*)) = ((x, a)^*,\tilde{\mathfrak{L}}((x, a)^*)).$$

Definition 2.9: [14] Let $(U, A, \tilde{I})$ be a fuzzy formal context. For an attribute $a \in A$, the fuzzy attribute granular concept $\tilde{\epsilon\varepsilon}_a$ induced by attribute a is defined as follows:

$$\tilde{\epsilon\varepsilon}_a = \bigcup_{x \in U} ((x, a)^*,\tilde{\mathfrak{L}}((x, a)^*)\text{-}(\emptyset, A)). \tag{3}$$

Thus, the fuzzy attribute granular concept space is given by:

$$\tilde{\epsilon\varepsilon} = \bigcup_{j=1}^{m} \tilde{\epsilon\varepsilon}_{a_j}.$$

The fuzzy granular concept space Q_k, corresponding to the decision class $c_k(k \leq l)$, is expressed as

$$Q_k = \epsilon\varepsilon_k \bigcup \tilde{\epsilon\varepsilon}.$$

III. CONSTRUCT THE γ-FUZZY GRANULAR CONCEPT SPACE

In a multi-decision context, the weight distribution of attributes across different decision classes may vary. By adjusting the attribute weights according to their relevance in each decision class, complex information can be more effectively managed. This section discusses the weighting of attributes based on the fluctuations in the degree of membership. We quantify the contribution of each concept by analyzing these fluctuations and then retain the concepts with higher contributions to reduce redundancy in the concept space, thereby enhancing the efficiency of CCL.

Definition 3.1: Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context, where the correlation of attribute $a_j \in A$ with the decision class c_k is as follows:

$$\delta_{c_k}(a_j) = \frac{1}{|c_k|} \sum_{x_i \in c_k} \tilde{I}(x_i, a_j), \tag{4}$$

where $\delta_{c_k}(a_j)$ represents the average correlation between attribute a_j and the decision class c_k. While this reflects the general association between attributes and decisions, it does not capture the relative importance of each attribute in making decisions. Next, we measure the fluctuation of attribute values under different decision classes to better understand how an attribute varies in importance. The greater the fluctuation, the smaller the attribute's weight.

Definition 3.2: Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context. The fluctuation of attribute $a_j \in A$ with respect to decision class c_k is given by:

$$Fl_{c_k}(a_j) = \sum_{x_i \in c_k} |\tilde{I}(x_i, a_j) - \delta_{c_k}(a_j)|. \tag{5}$$

The weight of attribute a_j in decision class c_k is as follows:

$$w_{kj} = \begin{cases} \frac{1}{m-1} - \frac{Fl_{c_k}(a_j)}{(m-1)\sum\limits_{h=1}^{m} Fl_{c_k}(a_h)}, & \sum\limits_{h=1}^{m} Fl_{c_k}(a_h) \neq 0, \\ \frac{1}{m}, & \text{otherwise}, \end{cases} \tag{6}$$

where m represents the total number of attributes. The weight matrix w is then formed as:

$$w = \begin{pmatrix} w_{11} & w_{12} & \ldots & w_{1m} \\ w_{21} & w_{22} & \ldots & w_{2m} \\ \vdots & \vdots & & \vdots \\ w_{l1} & w_{l2} & \ldots & w_{lm} \end{pmatrix},$$

where w_{kj} indicates the weight of a_j in decision class c_k.

Definition 3.3: Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context. If $(X,\tilde{B})$ is a concept in the fuzzy decision granular concept space Q_k, then the concept contribution degree of $(X,\tilde{B})$ is as follows:

$$CCD_d(X,\tilde{B}) = 1 - \sum_{j=1}^{m} |\delta_{c_k}(a_j) - \tilde{B}(a_j)| w_{kj}. \tag{7}$$

In the fuzzy decision granular concept space, concepts reflect their degree of influence on decision classes through their contribution degree. Concepts with higher contribution degrees are more prominent within the concept space. By selecting the concepts with higher contribution degrees, the redundancy in the concept space is reduced, and the influence of irrelevant and noisy attributes is effectively reduced, leading to improved learning efficiency.

Definition 3.4: Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context, and let γ (where $0 \leq \gamma \leq 1$) be a threshold. The γ-fuzzy granular concept space Q_k^{γ} is as follows:

$$Q_k^{\gamma} = \{(X,\tilde{B}) \in Q_k | CCD_d(X,\tilde{B}) \geq \gamma\}.$$

The threshold γ serves as a critical value for the concept contribution degree. It helps to filter out concepts with higher contribution values and controls the number of concepts in the concept space. If the threshold is too high, important concepts may be excluded; if it is too low, the concept space may become excessively large and redundant. Therefore, γ should be adjusted and optimized according to the specific task and domain requirements. Algorithm 1 presents the construction process for the γ-fuzzy granular concept space Q_k^{γ}.

Example 3.1: In real medical scenarios, disease diagnosis requires evaluation through multiple indicators. The pneumonia diagnosis dataset of the respiratory department of a certain hospital, as shown in Table I, contains 4 key symptom indicators (a_1, a_2, a_3, a_4) of 6 suspected pneumonia patients $(x_1, x_2, x_3, x_4, x_5, x_6)$, which are specifically mapped to body temperature and blood oxygen saturation. The specific mappings are body temperature, blood oxygen saturation, white blood cell count and lung CT shadows. The final diagnosis results $(d_1 = 1,\ d_2 = 0)$ and $(d_1 = 0,\ d_2 = 1)$ represent non-pneumonia and pneumonia respectively. The patients were divided into 2 decision classes as follows:

$$c_1 = \{x_1, x_2, x_3\}, c_2 = \{x_4, x_5, x_6\}.$$

TABLE I
FUZZY DECISION FORMAL CONTEXT.

U	a_1	a_2	a_3	a_4	d_1	d_2
x_1	0.22	0.62	0.07	0.04	1	0
x_2	0.19	0.58	0.10	0.13	1	0
x_3	0.22	0.58	0.08	0.04	1	0
x_4	0.39	0.25	0.42	0.38	0	1
x_5	0.42	0.25	0.51	0.46	0	1
x_6	0.58	0.29	0.73	0.75	0	1

Fuzzy granular concept space:

$$\begin{aligned} Q_1 = \{ & (\{x_1\}, \{0.22, 0.62, 0.07, 0.04\}); \\ & (\{x_2\}, \{0.19, 0.58, 0.10, 0.13\}); \\ & (\{x_3\}, \{0.22, 0.58, 0.08, 0.04\}); \\ & (\{x_6\}, \{0.58, 0.29, 0.73, 0.75\}); \\ & (\{x_5, x_6\}, \{0.42, 0.25, 0.51, 0.46\}); \\ & (\{x_1, x_2, x_3\}, \{0.19, 0.58, 0.07, 0.04\}); \\ & (\{x_4, x_5, x_6\}, \{0.39, 0.25, 0.42, 0.38\}); \\ & (\{x_1, x_2, x_3, x_6\}, \{0.19, 0.29, 0.07, 0.04\}); \\ & (\{x_2, x_4, x_5, x_6\}, \{0.19, 0.25, 0.10, 0.13\}); \\ & (\{x_1, x_3, x_4, x_5, x_6\}, \{0.22, 0.25, 0.07, 0.04\}); \\ & (\{x_2, x_3, x_4, x_5, x_6\}, \{0.19, 0.25, 0.08, 0.04\})\}, \end{aligned}$$

$$\begin{aligned} Q_2 = \{ & (\{x_1\}, \{0.22, 0.62, 0.07, 0.04\}); \\ & (\{x_6\}, \{0.58, 0.29, 0.73, 0.75\}); \\ & (\{x_5, x_6\}, \{0.42, 0.25, 0.51, 0.46\}); \\ & (\{x_1, x_2, x_3\}, \{0.19, 0.58, 0.07, 0.04\}); \\ & (\{x_4, x_5, x_6\}, \{0.39, 0.25, 0.42, 0.38\}); \\ & (\{x_1, x_2, x_3, x_6\}, \{0.19, 0.29, 0.07, 0.04\}); \\ & (\{x_2, x_4, x_5, x_6\}, \{0.19, 0.25, 0.10, 0.13\}); \\ & (\{x_1, x_3, x_4, x_5, x_6\}, \{0.22, 0.25, 0.07, 0.04\}); \\ & (\{x_2, x_3, x_4, x_5, x_6\}, \{0.19, 0.25, 0.08, 0.04\})\}. \end{aligned}$$

The following text uses $C_1 \sim C_{20}$ to represent the above 20 concepts in sequence.

Under different decision classes, calculate the correlation of attributes with respect to decision classes.

$$\delta_{c_1}(a_1) = 0.21,\ \delta_{c_1}(a_2) = 0.59,$$
$$\delta_{c_1}(a_3) = 0.08,\ \delta_{c_1}(a_4) = 0.07,$$
$$\delta_{c_2}(a_1) = 0.46,\ \delta_{c_2}(a_2) = 0.26,$$
$$\delta_{c_2}(a_3) = 0.55,\ \delta_{c_2}(a_4) = 0.53.$$

Compute attribute fluctuation:

$$Fl_{c_1}(a_1) = 0.04, Fl_{c_1}(a_2) = 0.05,$$
$$Fl_{c_1}(a_3) = 0.03, Fl_{c_1}(a_4) = 0.12,$$
$$Fl_{c_2}(a_1) = 0.23, Fl_{c_2}(a_2) = 0.05,$$
$$Fl_{c_2}(a_3) = 0.35, Fl_{c_2}(a_4) = 0.44.$$

Weight the attributes:

$$w = \begin{pmatrix} 0.28 & 0.26 & 0.29 & 0.17 \\ 0.26 & 0.32 & 0.22 & 0.20 \end{pmatrix}.$$

The contribution degree of all concepts in the fuzzy granular concept space are as follows:

$$CCD_1(C_1) = 0.92,\quad CCD_1(C_2) = 0.89,$$
$$CCD_1(C_3) = 0.95,\quad CCD_1(C_4) = -1.00,$$
$$CCD_1(C_5) = -0.37,\quad CCD_1(C_6) = 0.93,$$
$$CCD_1(C_7) = -0.19,\quad CCD_1(C_8) = 0.64,$$
$$CCD_1(C_9) = 0.56,\quad CCD_1(C_{10}) = 0.61,$$
$$CCD_1(C_{11}) = 0.61,\ CCD_2(C_{12}) = -0.57,$$
$$CCD_2(C_{13}) = 0.45,\ CCD_2(C_{14}) = 0.84,$$
$$CCD_2(C_{15}) = -0.02,\ CCD_2(C_{16}) = 0.62,$$
$$CCD_2(C_{17}) = -0.27,\ CCD_2(C_{18}) = 0.13,$$

Algorithm 1: Construct the γ-fuzzy granular concept space Q_k^{γ}.

Input: Fuzzy formal decision context $(U, A, \tilde{I}, D, J)$ and threshold γ.
Output: γ-fuzzy granular concept space Q_k^{γ}.

```
begin
    Compute U/D = { c_1, c_2, ..., c_l }.
    for k = 1 : l do
        Q_k^γ ← ∅.
        Compute Q_k.
        for j = 1 : |A| do
            Compute Fl_{c_k} and w_{kj}.
        end
        for (X, B̃) ∈ Q_k do
            Compute CCD_d(X, B̃).
        end
        if C_d(X, B̃) ≥ γ then
            Q_k^γ ← (X, B̃).
        end
    end
    return {Q_1^γ, Q_2^γ, ..., Q_l^γ}.
end
```

$CCD_2(C_{19})$ = -0.22, $CCD_2(C_{20})$ = -0.24.

Let the threshold $\gamma = 0.6$. The γ-fuzzy granular concept space is as follows:

$$Q_1^{\gamma} = \{C_1, C_2, C_3, C_6, C_8, C_{10}, C_{11}\},$$
$$Q_2^{\gamma} = \{C_{14}, C_{16}\}.$$

IV. CONCEPT SPACE CLUSTERING AND LABEL PREDICTION

The previous section addressed the construction of the γ-fuzzy granular concept space. Although the threshold γ provides a means to control the number of concepts within the space, it ensures that important concepts are not inadvertently excluded. However, this results in a relatively large number of concepts, which significantly reduces the computational efficiency of the fuzzy CCL model. To mitigate this issue, the present paper proposes a method to measure the degree of concept correlation of fusion extent and intent. The concept clustering of dynamic fusion is used to generate pseudo-concepts and reduce the scale of the γ-fuzzy granular concept space, so as to avoid the problems of unstable quality of pseudo-concepts and dilution of key information caused by fixed fusion sequence.

Definition 4.1: Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context, and let Q_k^{γ} denote the γ-fuzzy granular concept space. If $(X, \tilde{B}) \in Q_k^{\gamma}$ and for every $(X', \tilde{B}') \in Q_k^{\gamma}$, it holds that $CCD_d(X, \tilde{B}) \geq CCD_d(X', \tilde{B}')$, then $(X, \tilde{B})$ is called a key concept within the γ-fuzzy granular concept space.

In the context of concept clustering, the degree of correlation between concepts serves as a measure of the rationality of the clustering process. We introduce a method for evaluating the degree of concept correlation that considers both the extent and the intent of the concepts.

Definition 4.2: Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context, and let Q_k^{γ} be the corresponding γ-fuzzy granular concept space. The degree of correlation between two concepts, $(X_1, \tilde{B}_1)$ and $(X_2, \tilde{B}2) \in Q_k^{\gamma}$, is as follows:

$$\theta_{1,2} = \frac{\partial_5 \left|X_1 \bigcap X_2\right|}{\partial_5 \left|X_1 \bigcap X_2\right| + \partial_3 \left|X_1 - X_2\right| + \partial_4 \left|X_2 - X_1\right|}, \quad (8)$$

where, $\partial_1 = CCD_d(X_1, \tilde{B}_1)$, $\partial_2 = CCD_d(X_2, \tilde{B}_2)$, $\partial_3 = \frac{|X_1 - X_2|}{|X_1|}\partial_1$, $\partial_4 = \frac{|X_2 - X_1|}{|X_2|}\partial_2$, $\partial_5 = \left|X_1 \bigcap X_2\right| \left(\frac{\partial_1}{|X_1|} + \frac{\partial_2}{|X_2|}\right)$.

A higher value of $\theta_{1,2}$ indicates a stronger correlation between the two concepts. Let $(U_s, A, \tilde{I}_s, D, J_s)$ be a sub-context of $(U, A, \tilde{I}, D, J)$. The clustering process for its γ-fuzzy granular concept space proceeds as follows: First, the concepts are sorted by their contribution degrees in descending order, with the key concept $(X_i, \tilde{B}_i)$ serving as the core of the clustering. Next, the degree of correlation, $\theta_{i,j}$, between $(X_i, \tilde{B}_i)$ and each remaining concept $(X_j, \tilde{B}_j)$ is computed. If $\theta_{i,j} \geq \eta$ for a given threshold η (where $0 \leq \eta \leq 1$), then $(X_j, \tilde{B}_j)$ and $(X_i, \tilde{B}_i)$ are considered as belonging to the same pseudo-concept cluster Q^p and are removed from the concept space. This clustering process is repeated until no further concepts remain in the space. The generated pseudo-concept clusters are then clustered to form new pseudo-concepts.

Definition 4.3: Let $(U, A, \tilde{I}, D, J)$ be a fuzzy formal decision context. For any pseudo-concept cluster $Q_k^p = \{(X_1, \tilde{B}_1), (X_2, \tilde{B}_2), \ldots, (X_p, \tilde{B}_p)\}$, the corresponding generated pseudo-concepts are as follows:

$$X^p = X_1 \bigcup X_2 \bigcup \ldots \bigcup X_p,$$
$$\tilde{B}^p(a) = \frac{1}{2^{p-1}}(\tilde{B}_1(a) + \tilde{B}_2(a) + 2\tilde{B}_3(a) \ldots + 2^{p-2}\tilde{B}_p(a)).$$

Algorithm 2 outlines the clustering process for the γ-fuzzy granular concept space. Upon completing concept clustering within the γ-fuzzy granular concept space, the γ-fuzzy granular concept clustering space, denoted as Q^F, is obtained as follows:

$$Q^F = \begin{bmatrix} Q_1^F \\ Q_2^F \\ \vdots \\ Q_l^F \end{bmatrix} = \begin{bmatrix} (X_1^{p\,1}, \tilde{B}_1^{p\,1}) & (X_1^{p\,2}, \tilde{B}_1^{p\,2}) & \ldots & (X_1^{p\,\omega_1}, \tilde{B}_1^{p\,\omega_1}) \\ (X_2^{p\,1}, \tilde{B}_2^{p\,1}) & (X_2^{p\,2}, \tilde{B}_2^{p\,2}) & \ldots & (X_2^{p\,\omega_2}, \tilde{B}_2^{p\,\omega_2}) \\ \vdots & \vdots & & \vdots \\ (X_l^{p\,1}, \tilde{B}_l^{p\,1}) & (X_l^{p\,2}, \tilde{B}_l^{p\,2}) & \ldots & (X_l^{p\,\omega_l}, \tilde{B}_l^{p\,\omega_l}) \end{bmatrix},$$

where $\omega_k = \left|Q_k^F\right| (1 \leq k \leq l)$.

Example 4.1: (Continued from Example 3.1) Let the threshold be $\eta = 0.65$. The γ-fuzzy granular concept space Q_1^{γ} is divided into three clusters as follows:

$$\{C_3, C_6, C_8, C_{10}, C_{11}\}, \{C_1\}, \{C_2\}.$$

Similarly, the division of the γ-fuzzy granular concept space Q_2^{γ} is as follows:

$$\{C_{14}, C_{16}\}.$$

For the pseudo-concept clusters $\{C_3, C_6, C_8, C_{10}, C_{11}\}$ and $\{C_{14}, C_{16}\}$, the corresponding generated pseudo-concepts $C_3^{'}$ and $C_{14}^{'}$ are as follows:

$$(\{x_2, x_3, x_4, x_5, x_6\}, \{0.20, 0.39, 0.07, 0.04\});$$
$$(\{x_4, x_5, x_6\}, \{0.41, 0.25, 0.47, 0.42\}).$$

Algorithm 2: Construct the γ-fuzzy granular concept clustering space Q^F.

Input: γ-fuzzy granular concept space Q_k^{γ} and threshold η.
Output: γ-fuzzy granular concept clustering space Q^F.
begin
  for $k = 1 : l$ **do**
    $Q_k^F \leftarrow \varnothing$.
    while $Q_k^{\gamma} \neq \varnothing$ **do**
      Sort the concepts in Q_k^{γ}.
      $Q^p \leftarrow (X_1, \tilde{B}_1)$, $Q_k^{\gamma} \leftarrow (X_1, \tilde{B}_1)$.
      for $j = 2 : \left|Q_k^{\gamma}\right|$ **do**
        if $X_1 \bigcap X_j \neq \varnothing$ **then**
          Compute $\theta_{1,j}$.
          if $\theta_{1,j} \geq \eta$ **then**
            $Q^p \leftarrow (X_j, \tilde{B}_j)$, $Q_k^{\gamma} \leftarrow (X_j, \tilde{B}_j)$.
          end
        end
      end
      for $(X_j, \tilde{B}_j) \in Q^p$ **do**
        Generate $(X^p\, \tilde{B}^p)$.
        $Q_k^F \leftarrow (X^p\, \tilde{B}^p)$.
      end
    end
  end
  return $\left\{Q_1^F, Q_2^F, \ldots, Q_l^F\right\}$.
end

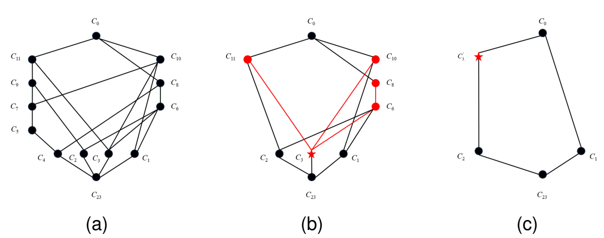


Fig. 1. A graphical demonstration for a new fuzzy concept generation.

In order to represent the processes of concept generation, Fig. 1 illustrates the overall procedure of a new fuzzy concept generation, where $C_0 = (U,\ \varnothing)$ and $C_{23} = (\varnothing,\ A)$. Meanwhile, the processes of concept generation can be represented as follows. Firstly, based on Algorithm 1, a fuzzy granular concept space can be constructed (see Fig. 1(a) for details). Secondly, construct γ-fuzzy granular concept space, select the key concept as clustering core., and then we can generate a fuzzy concept by Definition 4.3 (see Fig. 1(b) for further details). Finally, in a similar manner, a pseudo-concept can be constructed, and we can obtain a new γ-fuzzy granular concept clustering space and its corresponding fuzzy concepts. Please refer to Fig. 1(c) for details.

In practical applications, existing fuzzy CCL models typically perform label prediction by similarity matching after clustering or weighting the fuzzy concepts. However, this approach only considers the similarity between concepts and neglects the structural information inherent among the fuzzy concepts. To enhance the performance of label prediction, it is crucial to develop a model that incorporates both similarity and structural information. Set approximation [21] serves as an effective approach for CCL, leveraging available clues. Through the upper and lower approximations, more abundant information is extracted from the hierarchical relationship between the sub-concepts and the parent concepts. This effectively solves the problem that the traditional similarity matching model only relies on the surface similarity between concepts and ignores the structural information. Consequently, it becomes necessary to construct upper and lower approximation spaces based on the clustered γ-fuzzy granular concept space.

Definition 4.4: Let Q_k^F represent the γ-fuzzy granular concept clustering space. For any fuzzy set $\tilde{B}_t$, the upper and lower approximation spaces of $\tilde{B}_t$ in the γ-fuzzy granular concept clustering space Q^F are as follows:

$$\overline{Aprh_k}(\tilde{B}_t) = \bigcup_{(X,\tilde{B})\in Q_k^F,\, \tilde{B}\subset \tilde{B}_t} \tilde{B},$$

$$\underline{Aprl_k}(\tilde{B}_t) = \bigcap_{(X,\tilde{B})\in Q_k^F,\, \tilde{B}_t\subseteq \tilde{B}} \tilde{B},$$

where $(\overline{Aprh_1}(\tilde{B}_t),\ \overline{Aprh_2}(\tilde{B}_t), \ldots, \overline{Aprh_l}(\tilde{B}_t))$ and $(\underline{Aprl_1}(\tilde{B}_t),\ \underline{Aprl_2}(\tilde{B}_t), \ldots, \underline{Aprl_l}(\tilde{B}_t))$ denote the upper and lower approximation spaces of $\tilde{B}_t$ under the γ-fuzzy granular concept clustering space Q^F, respectively.

Label prediction is performed by calculating the maximum similarity between the fuzzy set of the testing sample and the fuzzy sets in the upper and lower approximation spaces:

$$\overline{SAh_k}(\tilde{B}_t) = 1 - \sqrt{\sum_{j=1}^{m} \left(\tilde{B}_t(a_j) - \overline{Aprh_k}(\tilde{B}_t(a_j))\right)^2},$$

$$\underline{SAl_k}(\tilde{B}_t) = 1 - \sqrt{\sum_{j=1}^{m} \left(\tilde{B}_t(a_j) - \underline{Aprl_k}(\tilde{B}_t(a_j))\right)^2}.$$

Thus, the label prediction is determined as: $label = \arg\ \max(\max(\underline{SAl_{k_1}}),\ \max(\overline{SAh_{k_2}}))$, where $k_1,\ k_2 = (1,\ 2, \ldots,\ l)$. Obviously, the above maximum similarities not only consider the similarity between fuzzy granular concepts, but also include the structure information among them. Algorithm 3 presents the label prediction process for the testing samples.

Fig. 2 illustrates the overall framework of the proposed Attribute Fluctuation-Based CCL (AFFCCL) model, which consists of three stages:

(1) Construction of the fuzzy granular concept space.

(2) Formation of the γ-fuzzy granular concept space Q_k^{γ} and the γ-fuzzy granular concept clustering space Q^F based on attributes fluctuation, threshold γ and η.

(3) Establishment of the upper and lower approximation spaces for label prediction of testing samples.

The time complexity of the AFFCCL is determined by the time complexities of Algorithm 1 to 3. In Algorithm 1, the time complexity of constructing the γ-fuzzy granular concept space $O(|U|\,(|U|\,|A|+|D|))$. In Algorithm 2, the time

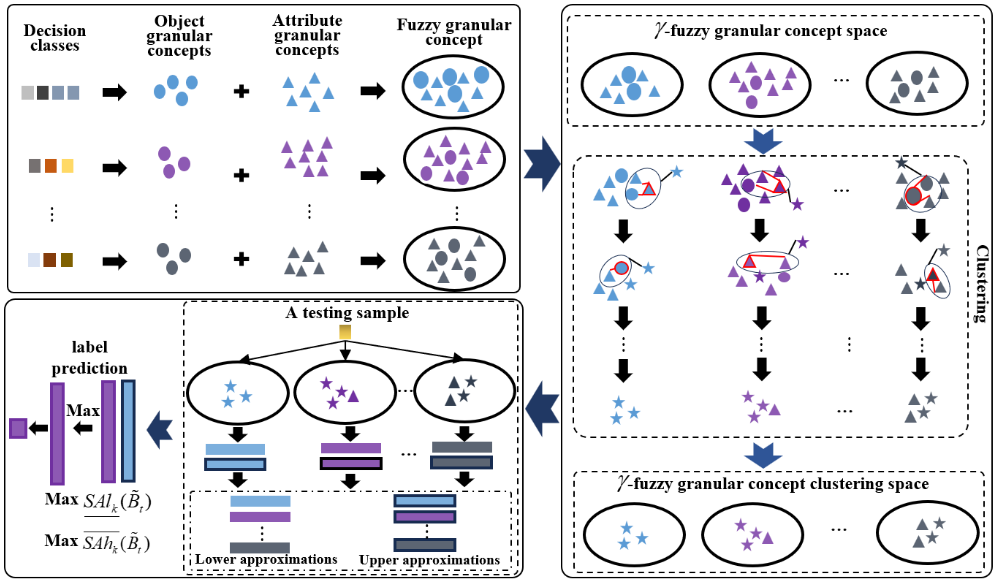


Fig. 2. Overall framework of AFFCCL.

Algorithm 3: Label prediction of testing samples.

Input: Testing samples x_t and its $\tilde{B}_t$, γ-fuzzy granular concept clustering space $Q^F = \{Q_1^\gamma, Q_2^\gamma, \ldots, Q_I^\gamma\}$.
Output: The label l_t for testing samples x_t.
begin
  Compute $(\overline{Aprh_1}(\tilde{B}_t), \overline{Aprh_2}(\tilde{B}_t), \ldots, \overline{Aprh_l}(\tilde{B}_t))$ and $(\underline{Aprl_1}(\tilde{B}_t), \underline{Aprl_2}(\tilde{B}_t), \ldots, \underline{Aprl_l}(\tilde{B}_t))$.
  $LD \leftarrow \varnothing$, $UD \leftarrow \varnothing$.
  for $k = 1 : l$ **do**
    Compute $\overline{SAh_k}(\tilde{B}_t)$ and $\underline{SAl_k}(\tilde{B}_t)$.
    $UD \leftarrow \overline{SAh_k}(\tilde{B}_t)$, $LD \leftarrow \underline{SAl_k}(\tilde{B}_t)$.
  end
  Compute $l = \arg\ \max(\max(UD),\ \max(LD))$.
  return l_t.
end

complexity of constructing the γ-fuzzy granular concept clustering space $O(|U|\,|A|^2)$. In Algorithm 3, the time complexity of label prediction for testing samples is $O(|U|\,|A|^2\,|D|)$. Therefore, the overall time complexity of the AFFCCL model is $O(|U|\,|A|^2\,|D|)$.

V. EXPERIMENTAL STUDY

In this section, experiments are conducted on 15 UCI datasets, and comparisons are made with multiple existing methods to assess the performance of the proposed AFFCCL model.

A. General Settings

To evaluate the classification performance of the proposed AFFCCL model, 15 datasets were downloaded from the UCI datasets. Detailed information about these datasets is presented in Table II. All datasets underwent Min-Max normalization to generate fuzzy datasets $(U, A,\ \tilde{I}, D,\ J)$. The normalized value $\tilde{I}(x,\ a_j)$ for object x under conditional attribute a_j is defined as follows:

$$\tilde{I}(x,\ a_j) = \frac{f(x,\ a_j) - \min\limits_{1\le i\le n} f(x_i,\ a_j)}{\max\limits_{1\le i\le n} f(x_i,\ a_j) - \min\limits_{1\le i\le n} f(x_i,\ a_j)},$$

where $f(x,\ a_j)$ represents the initial value of object x under conditional attribute a_j, $\min f(x,\ a_j)$ and $\max f(x,\ a_j)$ denote the minimum and maximum values of all objects under conditional attribute a_j, respectively. In the experiments, 80% of the data from each dataset was randomly selected as the training set for concept prediction, while the remaining 20% was used for testing.

TABLE II
DATA SET INFORMATION TABLE.

ID	Data set	Object	Attribute	Class
1	Molecular Biology	106	57	2
2	Iris	150	4	3
3	Hepatitis	155	19	2
4	Planning	182	13	2
5	Parkinsons	197	23	2
6	Sonar	208	60	2
7	Glass	214	9	6
8	Audiology	226	69	18
9	Spectf Heart	267	23	2
10	Breast	286	9	2
11	Haberman's Survival	306	3	2
12	Dermatology	366	34	6
13	Indian Liver Patient	583	10	2
14	Tic-Tac-Toe	958	9	2
15	Image Segmentation	2310	19	7

To verify the effectiveness and feasibility of the proposed model, AFFCCL was compared with 20 other classification methods. These include 13 fuzzy similarity-based classification methods, such as CFKNN [22], FENN [23], FRNN [24], and others, as well as 6 fuzzy CCL-based classification methods, including FCLM [16], DMPWFC [25], FRCM [26], MFCCL [17], FGCCL [14], and AFFCCLCS (AFFCCL+ Cosine Similarity measure method), as well as a neural network classification method, MLP (Multilayer Perceptron).

Additionally, to improve the model's adaptability to real-world complexity and enhance robustness, Gaussian perturbation was introduced into the concept contribution calculation. as follows:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}.$$

Specifically, let μ represent the mean (location parameter), σ^2 the variance (scale parameter), and e a random variable. To minimize statistical errors, all experiments were repeated 10 times. The experiments were performed on a platform running Windows 10, with an Intel(R) Core(TM) i5-6500 CPU @ 3.2 GHz and 8GB of memory, using Python 3.6 as the development environment.

B. Comparison of the Number of Concepts

This section analyzes the impact of varying thresholds on the number of concepts generated by the AFFCCL model across the 15 datasets. The change in the number of concepts is illustrated in Fig. 3, where Initial Concept Count DFk, K Concept Count DFk and Clustered Concept Count DFk $(k = 1, 2, \ldots, l)$ respectively represent the number of concepts in fuzzy granular concept space Q_k, γ-fuzzy granular concept space Q_k^{γ} and γ-fuzzy granular concept clustering space Q_k^F. Since the number of concepts generated and the threshold value ranges differ across datasets, the plots in Fig. 3 exhibit different characteristics. As shown, the number of concepts in the fuzzy granular concept space decreases monotonically as the threshold γ increases, while the number of concepts in γ-fuzzy granular concept space increases monotonically as the threshold η increases. This behavior effectively reduces model complexity.

C. Threshold Analysis

In the experiment, both thresholds γ and η range from [0, 1], with a step size of 0.01. For the threshold γ, if the value is too large, all concepts are discarded, resulting in a sharp decline in classification accuracy. Conversely, if γ is too small, it has negligible effects on classification accuracy and model complexity. When $\gamma \leq 0.40$, the classification accuracy remains almost unchanged, but when $\gamma \geq 0.95$, no output is generated, resulting in zero classification accuracy. Therefore, the threshold γ was set within the range [0.40, 0.95] with a step size of 0.01. The threshold η is influenced by γ. When $\eta \geq 0.80$, its impact on classification accuracy becomes more pronounced. Consequently, γ as set within the range [0.80, 0.99], also with a step size of 0.01. Fig. 4 shows the impact of threshold variations on the classification accuracy of AFFCCL.

In datasets (a), (c), (d), (f), (l), and (o), the classification accuracy is primarily influenced by changes in γ, with minimal sensitivity to η. These thresholds mainly affect the size of the concept space, reducing model complexity while maintaining classification accuracy. In contrast, datasets (c), (d), (l), and (o) demonstrate improvements in classification accuracy as η changes, while also reducing model complexity. The classification accuracy of datasets (b), (e), (g), and (j) is influenced by both thresholds, but datasets (h), (i), (k), (m), and (n) show more significant sensitivity to η. These results suggest that the thresholds have varying degrees of importance, with γ primarily affecting the number of concepts and model complexity, and η mainly affecting classification accuracy. In conclusion, AFFCCL can strike a balance between complexity and accuracy by selecting a reasonable threshold. The effectiveness of the multi-granularity mechanism and concept clustering in improving the classification results has been proved.

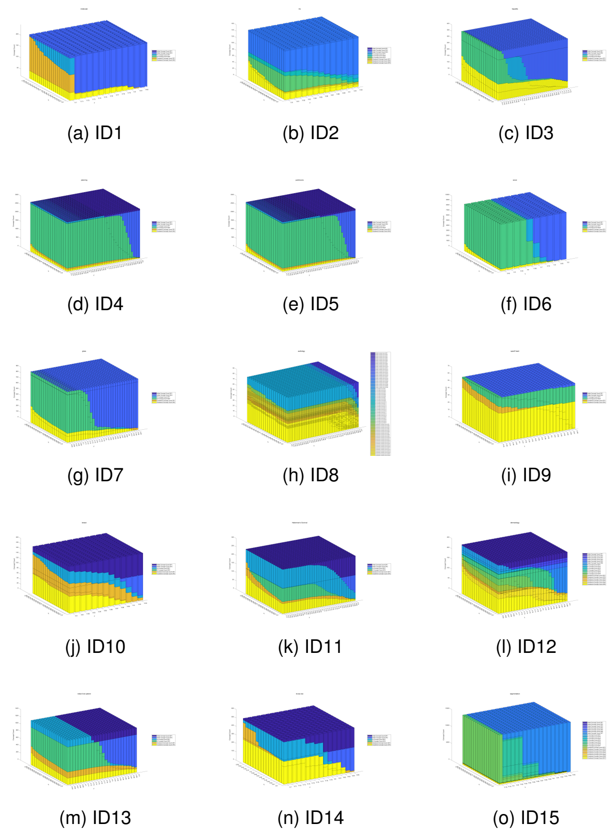


Fig. 3. Graph of Concept Count changes under different thresholds.

D. Comparison of Classification Performance

For comparison purposes, the algorithms are grouped into two categories. The first group includes 6 fuzzy CCL algorithms and a neural network algorithm, the second group includes 13 fuzzy similarity-based classification algorithms. The classification performance of AFFCCL, along with the two groups of algorithms, is summarized in Table III. The best accuracy for each dataset is highlighted in bold. From Table III, it is evident that AFFCCL achieves an average classification accuracy of 87.32%, which is 4.61% higher than FCLM, 11.78% higher than DMPWFC, 5.48% higher than FRCM, 18.94% higher than MFCCL , 1.24% higher than

TABLE III
AVERAGE PREDICTION ACCURACY (MEAN ± STANDARD DEVIATION %) OF AFFCCL AND 17 CLASSIFICATION METHODS BASED ON FUZZY CCL AND FUZZ SIMILARITY.

ID	AFFCCL	FCLM	DMPWFC	FRCM	MFCCL	FGCCL	AFFCCLCS	MLP
ID1	**86.82±6.25**	73.64±6.36	43.31±12.81	76.15±7.86	59.50±0.81	84.25±1.13	68.45±7.61	65.91±11.03
ID2	**100.00±0.00**	97.67±1.53	96.96±2.29	96.00±2.63	58.37±2.10	97.67±2.60	91.25±3.96	97.67±3.00
ID3	**90.67±2.49**	81.94±5.24	69.97±4.98	84.86±5.40	84.57±2.41	89.71±2.62	83.87±5.45	90.00±7.04
ID4	**73.55±3.76**	68.11±2.02	60.28±10.57	62.89±6.38	54.73±2.72	72.21±1.58	62.93±5.90	67.31±3.24
ID5	**97.50±1.94**	96.92±1.54	55.57±16.15	91.86±4.28	79.07±3.74	96.97±0.73	90.06±2.54	97.21±1.71
ID6	87.38±4.13	86.90±3.06	81.89±13.51	84.38±4.74	58.96±4.17	87.16±2.26	77.74±7.87	84.62±5.41
ID7	**83.26±5.58**	74.19±4.35	72.60±6.35	70.87±6.50	60.45±6.22	77.83±4.54	80.64±4.45	78.53±3.89
ID8	82.39±2.47	75.50±4.85	80.91±10.70	75.23±3.78	56.06±6.45	**85.65±1.79**	62.21±2.41	79.65±5.42
ID9	**84.81±2.46**	80.93±4.06	73.56±4.68	77.29±2.90	84.07±1.93	80.14±3.62	76.47±1.53	81.46±4.61
ID10	77.93±1.69	72.76±6.01	77.61±9.11	78.39±7.42	64.52±4.52	**80.71±2.07**	66.82±6.64	73.08±6.33
ID11	**74.68±1.92**	72.42±5.92	73.18±4.61	68.79±5.21	73.99±2.15	73.04±3.27	66.19±4.95	74.02±3.93
ID12	96.76±1.83	96.08±1.41	93.15±4.85	96.79±1.09	83.30±1.57	95.19±1.31	95.31±2.10	95.49±1.36
ID13	**74.79±1.39**	67.01±2.87	62.48±2.41	66.97±3.76	72.18±2.10	74.12±1.12	62.83±4.05	66.64±5.73
ID14	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	65.10±1.75	**100.00±0.00**	83.90±1.93	**100.00±0.00**
ID15	**99.33±0.18**	96.58±0.90	91.64±5.38	97.16±0.73	70.85±2.47	96.49±0.73	97.86±1.91	96.95±2.31
Ave.	**87.32**	82.71	75.54	81.84	68.38	86.08	77.64	83.24

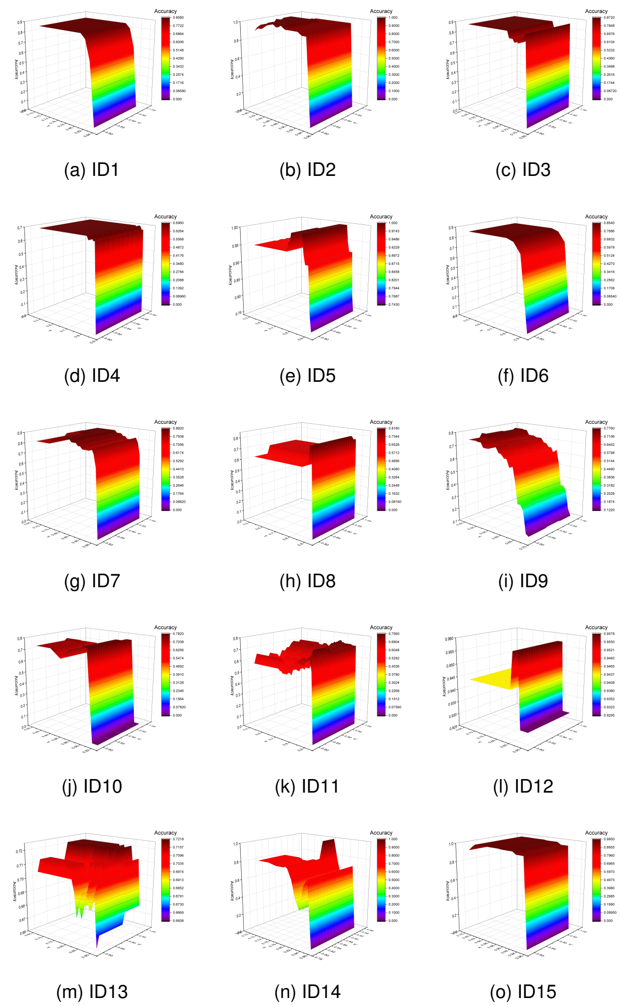
ID	AFFCCL	CFKNN	FENN	FRNN	FRNN-FRS	FRNN-VQRS	IF-KNN	PEKNN
ID1	**86.82±6.25**	79.36±13.01	82.18±11.01	73.55±12.94	83.65±1.77	82.08±1.96	82.27±13.62	80.92±1.43
ID2	**100.00±0.00**	94.00±8.14	95.33±4.27	88.67±7.33	95.33±4.27	95.33±4.27	94.00±4.67	86.67±14.30
ID3	**90.67±2.49**	85.00±12.25	86.25±10.38	88.75±6.63	41.25±14.84	75.00±13.69	86.25±10.38	83.75±11.25
ID4	**73.55±3.76**	55.92±2.23	67.05±4.10	71.46±1.52	64.77±8.45	64.24±7.68	67.60±8.08	68.74±9.23
ID5	**97.50±1.94**	74.31±0.90	93.68±0.89	84.84±0.78	94.89±4.53	94.47±0.92	93.95±5.82	91.28±1.20
ID6	87.38±4.13	68.21±11.98	77.79±8.97	86.48±6.18	**87.45±8.40**	86.48±7.77	81.17±10.31	77.31±10.72
ID7	**83.26±5.58**	45.42±11.41	68.48±13.17	62.48±8.76	70.86±11.80	69.84±10.54	69.99±11.99	71.09±12.99
ID8	82.39±2.47	60.79±11.25	60.24±9.11	56.21±9.69	63.79±11.52	57.71±11.60	63.32±12.57	58.26±12.46
ID9	**84.81±2.46**	69.02±8.30	65.90±4.67	69.73±6.87	50.48±7.92	47.08±7.56	62.93±5.79	60.71±5.62
ID10	77.93±1.69	67.80±7.26	73.44±4.97	71.37±6.08	72.39±9.16	70.32±7.29	70.25±5.46	67.80±6.89
ID11	**74.68±1.92**	65.06±6.02	70.26±5.18	73.53±0.94	64.04±5.65	64.04±5.65	71.55±5.02	67.01±7.65
ID12	96.76±1.83	96.42±0.38	96.73±2.02	94.29±0.54	95.35±3.26	94.41±0.51	96.93±0.37	75.44±9.83
ID13	**74.79±1.39**	63.64±4.60	65.21±5.66	71.36±0.72	65.52±6.49	65.35±6.37	66.55±4.32	61.40±5.81
ID14	**100.00±0.00**	70.19±0.53	76.62±0.20	69.93±0.61	72.00±0.34	71.92±0.20	79.08±0.47	76.42±0.50
ID15	**99.33±0.18**	75.67±1.79	94.11±1.22	81.90±1.28	95.19±0.94	95.19±0.94	94.72±1.00	78.10±1.66
Ave.	**87.32**	71.39	78.26	76.30	74.46	75.70	78.70	73.66

ID	AFFCCL	EF-KNN-IVFS	FRKNNA	FuzzyKNN	FuzzyNPC	GAFKNN	D-SKNN
ID1	**86.82±6.25**	71.72±13.80	75.64±10.58	82.27±13.62	78.45±14.80	80.36±12.64	82.27±13.62
ID2	**100.00±0.00**	94.67±4.99	92.67±4.67	94.00±4.67	92.67±7.57	94.00±4.67	94.00±4.67
ID3	**90.67±2.49**	87.50±7.91	86.25±8.75	83.75±12.56	83.75±12.56	81.25±10.08	85.00±10.90
ID4	**73.55±3.76**	61.58±11.62	57.11±5.63	69.18±8.04	55.49±1.89	68.57±13.31	65.94±7.31
ID5	**97.50±1.94**	95.56±1.07	88.89±1.50	94.95±5.48	76.41±1.06	96.47±0.67	93.94±5.82
ID6	87.38±4.13	79.29±6.58	80.67±9.61	84.05±9.02	68.69±10.07	85.98±7.45	82.62±9.51
ID7	**83.26±5.58**	64.33±9.23	63.84±10.06	72.25±12.10	48.43±12.27	73.28±12.05	71.00±11.48
ID8	82.39±2.47	56.13±6.35	56.19±7.73	72.50±11.78	76.97±10.07	70.50±8.71	65.87±9.17
ID9	**84.81±2.46**	64.86±5.35	64.87±6.54	60.68±5.60	67.85±10.32	61.82±5.03	61.05±6.07
ID10	77.93±1.69	66.08±5.27	68.18±7.59	68.53±9.51	67.45±8.12	67.86±7.75	67.14±7.11
ID11	**74.68±1.92**	68.26±5.70	71.55±4.29	67.33±5.81	65.72±6.58	68.98±6.76	69.94±5.05
ID12	96.76±1.83	95.96±0.65	94.96±0.76	**97.56±2.25**	96.46±2.98	97.23±0.25	95.67±3.86
ID13	**74.79±1.39**	64.52±4.82	61.75±6.56	66.21±3.67	64.66±5.27	64.85±5.30	65.69±3.59
ID14	**100.00±0.00**	80.22±0.52	84.24±0.60	79.12±0.46	68.51±0.53	80.24±0.47	79.12±0.46
ID15	**99.33±0.18**	93.16±1.38	93.07±1.75	95.50±0.93	75.32±1.97	94.68±1.06	94.85±1.26
Ave.	**87.32**	76.26	75.99	79.12	72.46	79.07	78.27

FGCCL, 9.68% higher than AFFCCLCS, and 4.08% higher than MLP. The AFFCCL model outperforms all algorithms in the first group. Except for the audiology (ID8), breast (ID10) and dermatology (ID12) datasets, AFFCCL achieves the highest accuracy on all other datasets. In comparison to the second group of algorithms, AFFCCL achieves the best accuracy on 13 of the 15 datasets, with the exception of the Sonar (ID6) and Dermatology (ID12) datasets.

E. Time Complexity Analysis

Through the time complexity analysis of AFFCCL model and 5 fuzzy CCL-based classification models, this section further analyzes that AFFCCL model achieves the best balance between time complexity and classification performance. The time complexity analysis is shown in Table IV, where, U_* is the updated object set, C is the granular reduction attribute set, $R(S)$ is the rule set, V is validation set, T is the test set, $C^{\prime S_{\widetilde{\lambda(i)}}}$ is the fuzzy concept clusters space, t, the number



(a) ID1 (b) ID2 (c) ID3

(d) ID4 (e) ID5 (f) ID6

(g) ID7 (h) ID8 (i) ID9

(j) ID10 (k) ID11 (l) ID12

(m) ID13 (n) ID14 (o) ID15

Fig. 4. Graph of Accuracy changes under different thresholds.

of iterations of the parameter γ_1 (usually 11), and w_k is the weighted fuzzy concept space.

TABLE IV
TIME COMPLEXITY OF CCL MODELS.

Model	Time complexity
AFFCCL	$O(\lvert U\rvert\,\lvert A\rvert^2\,\lvert D\rvert)$
MFCL	$O(\lvert U\rvert + t\,\lvert_k\rvert\,\lvert V\rvert + \lvert C^{S_{\widetilde{\lambda_1(i)}}}\rvert\,\lvert T\rvert)$
DMPWFC	$O(\lvert A\rvert\,(\lvert U\rvert + \lvert w_k\rvert^2))$
FRCM	$O(\lvert U_*\rvert\,\lvert A\rvert + \lvert U\rvert\,\lvert C\rvert + \lvert R(S)\rvert\,\lvert T\rvert)$
MFCCL	$O(\lvert U\rvert\,(\lvert U\rvert^2 + \lvert A\rvert))$
FGCCL	$O(\lvert U\rvert\,\lvert A\rvert^2\,\lvert D\rvert)$

The time complexity of both AFFCCL and FGCCL is $O(|U|^2|A|^2\,|D|)$. The size of $|U|$, $|A|$ and $|D|$ directly affects the construction time of concept space, which is determined by the size of data. It is suitable for medium-scale data, and the importance of attributes is uneven. The time complexity of FMCL is $O(|U| + t\,|_k|\,|V| + |C^{S_{\widetilde{\lambda_1(i)}}}|\,|T|)$. γ_1 Affects the number of clusters $|C^{S_{\widetilde{\lambda_1(i)}}}|$, the larger the γ_1, the fewer clusters and the faster the prediction stage, but it may reduce the accuracy. The size of $|U|$ directly affects the construction time of the concept space. The size of $|V|$and $|T|$ determines the time-consuming of the parameter tuning and prediction stage, which is specifically determined by the data size and the dominant factors of the stage. It is suitable for fuzzy concept clustering that requires parameter tuning. The time complexity of DMPWFC is $O(|A|\,(|U| + |w_k|^2))$. The size of $|U|$ and $|A|$ directly affects the construction time of the weighted fuzzy concept space. The size of the weighted fuzzy concept space $|w_k|$ affects the time-consuming of constructing the progressively weighted fuzzy concept space and the prediction stage. The time complexity of the DMPWFC model is determined by the size of the weighted fuzzy concept space. In the best case, the concepts generated by different objects are all the same, that is, $|w_k| = 1$. At this time, the total time complexity of the DMPWFC model is $O(|U|\,|A|)$. In the worst case, the concepts generated by different objects are all unique, that is, $|w_k| = |U|$, and the total time complexity of the DMPWFC model is $O(|U|^2\,|A|)$. Suitable for incremental learning under dynamic data. The time complexity of FRCM is $O(|U_*|\,|A| + |U|\,|C| + |R\,(S)|\,|T|)$. If $|R\,(S)|$ and $|T|$ are large, then $O(|R\,(S)|\,|T|)$ becomes the bottleneck. If $|U_*|$ and $|A|$ are large, then $O(|U_*|\,|A|)$ dominates. The time complexity of the FRCM model is determined by the fastest growing part of the data size. If the test phase dominates, the time complexity is $O(|R\,(S)|\,|T|)$. If the incremental learning phase dominates, the time complexity is $O(|U_*|\,|A|)$. Suitable for incremental learning or large-scale test sets. The time complexity of MFCCL is $O(|U|\,(|U|^2 + |A|))$. The size of $|U|$ and $|A|$ directly affects the construction time of concept space, which is determined by the data size. It is suitable for small-scale data.

In summary, AFFCCL achieves the best balance between time complexity and classification performance through weighted attributes, redundant filtering, and dynamic clustering. It is especially suitable for scenarios where attributes are of uneven importance and require real-time processing of incremental data. Its linear complexity make it perform well on medium-scale data, outperforming other models.

F. Non-parametric Test Analysis

To further validate the classification performance of each algorithm, the Friedman test was conducted to systematically assess whether the differences between AFFCCL and the 6 fuzzy CCL classification methods, MLP, as well as the 13 fuzzy similarity-based methods, are statistically significant. The test statistic F_F is expressed as:

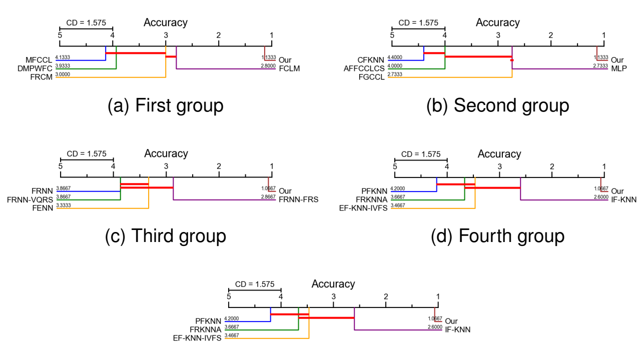
$$F_F = \frac{(N-1)\chi_F^2}{N(k-1) - \chi_F^2} \sim F(k-1, (k-1)(N-1)),$$

where $\chi_F^2 = \frac{12N}{k(k+1)}(\sum_{i=1}^{k} R_i^2 - \frac{k(k+1)^2}{4})$, N is the number of datasets, k is the number of algorithms, and R_i represents the average ranking of the i-th algorithm. Using $\alpha = 0.05$, the critical value of F_F is 1.608. The calculated F_F value of 9.797 exceeds the critical value, indicating significant performance differences. Subsequently, the Bonferroni-Dunn test was performed for further comparison. The critical range for the difference in average ranks is given by:

$$CD_\alpha = q_\alpha \sqrt{\frac{k(k+1)}{6N}}.$$

The algorithms were divided into five groups for comparison, and the Friedman test was conducted separately for each group. The results show that F_F values for each group exceed the critical values, indicating significant differences between the performance of AFFCCL and the other algorithms. The Bonferroni-Dunn test results confirm that AFFCCL significantly outperforms all other classification methods.

Since there are a large number of classification algorithms, the significance test needs to be strictly adjusted, resulting in a reduction in statistical power. Group comparison can reduce the magnitude of the adjustment and improve the sensitivity of the significance test. We evenly divide the algorithms into five groups for comparison with the AFFCCL classification method. To ensure the rigor of the experiment, the Friedman test was conducted on these four groups of classification methods respectively. When k=5, the critical value of the F_F test is $F_F = 2.537$. By calculating the five groups of classification methods, we can obtain F_{F1}=18.407, F_{F_2}=27.016, F_{F_3}=16.230, F_{F_4}=21.000, and F_{F_5}=18.950, all of which are greater than the critical values, indicating that there are differences in the performance of each group of algorithms. According to the critical value lookup table, when k=5, $q_{0.05} = 2.782$, $CD_\alpha = 1.575$. The Bonferroni-Dunn test diagrams of each group of classification methods are shown in Fig. 5. In each group of classification methods, the differences between the average ranking of the AFFCCL classification method and that of other classification methods are all greater than the critical values. From this, it can be known that the AFFCCL classification method is significantly different from other classification methods.



(a) First group (b) Second group

(c) Third group (d) Fourth group

(e) Fifth group

Fig. 5. Groups of Bonferroni-Dunn test chart of the algorithm.

G. Robustness and Consistency of AFFCCL Model

In order to further comprehensively verify the classification performance of AFFCCL and different classification methods on different data sets, robustness analysis will be carried out in this section. The robustness of any method $M_i (i = 1, 2, \ldots)$ on dataset ID$j (j = 1, 2, \ldots)$ is defined as the ratio of the

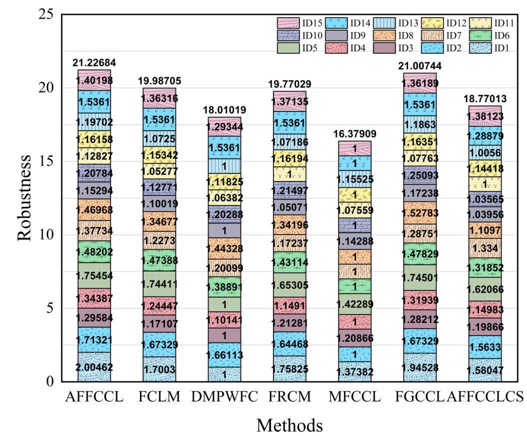


Fig. 6. Robustness analysis.

accuracy of method M_i on dataset IDj to the minimum accuracy achieved by all methods on the same dataset , and the definition of robustness is given as follows:

$$r_{M_i}(\mathrm{ID}j) = \frac{acc_{M_i}(\mathrm{ID}j)}{\min acc_{M_i}(\mathrm{ID}j)},$$

where $acc_{M_i}(\mathrm{ID}j)$ is the accuracy of method M_i on dataset IDj, and $\min acc_{M_i}(\mathrm{ID}j)$ is the minimum accuracy achieved by all methods on dataset IDj.

Therefore, the overall robustness of method M_i is the sum of its robustness across all datasets, denoted as r_{M_i}= $\sum_{j=1} r_{M_i}(\mathrm{ID}j)$. Additionally, it is important to note that a higher robustness value indicates better performance of the method. Fig. 6 presents a comparison of the robustness of AFFCCL with 6 fuzzy CCL-based classification methods across 15 datasets. From Fig. 6, it can be clearly observed that the overall robustness of AFFCCL ranks first among all the comparison algorithms, highlighting the robustness of the proposed method in classification tasks.

Finally, the Kappa coefficient was employed to evaluate the stability of the AFFCCL model and its capability to handle minority categories, providing a measure of the model's consistency. A higher Kappa coefficient indicates better consistency. Table V presents the average Kappa coefficients obtained through 10 repeated experiments on the 15 datasets. The results show that AFFCCL achieves higher Kappa coefficients compared to the other models, with an average coefficient that is 2% higher than FCLM, 13% higher than DMPWFC, 2% higher than FRCM, 19% higher than MFCCL, 0.1% higher than FGCCL, and 9% higher than AFFCCLCS. Moreover, the Kappa coefficient for AFFCCL on each dataset is at least 0.61, with 7 datasets achieving coefficients greater than 0.81, indicating high consistency in most cases.

VI. CONCLUSIONS

In response to the challenges of weak attribute-decision correlations, and redundancy in the concept space in existing CCL models, this paper proposes a novel AFFCCL model.

TABLE V
KAPPA COEFFICIENT OF CCL MODELS.

ID	AFFCCL	FCLM	DMPWFC	FRCM	MFCCL	FGCCL	AFFCCLCS
ID1	**0.82**	0.77	0.42	0.76	0.53	0.79	0.65
ID2	**1.00**	0.94	0.97	0.95	0.45	0.96	0.90
ID3	**0.70**	0.67	0.45	0.61	0.60	0.68	0.68
ID4	0.69	0.58	0.60	0.60	0.50	**0.71**	0.59
ID5	**0.90**	0.88	0.46	0.89	0.74	0.88	0.81
ID6	0.75	**0.76**	0.45	0.75	0.44	**0.76**	0.66
ID7	0.71	0.70	0.69	**0.73**	0.58	**0.73**	0.70
ID8	**0.83**	0.81	0.78	0.81	0.52	**0.83**	0.59
ID9	**0.65**	0.65	0.54	0.60	0.54	**0.65**	0.61
ID10	**0.73**	0.72	0.71	0.72	0.66	0.72	0.66
ID11	0.72	0.57	0.73	0.63	0.72	**0.74**	0.64
ID12	0.92	**0.96**	0.92	**0.96**	0.87	**0.96**	0.92
ID13	**0.61**	0.50	0.42	0.52	0.60	0.60	0.45
ID14	**1.00**	**1.00**	**1.00**	**1.00**	0.54	**1.00**	0.78
ID15	0.96	0.97	0.87	**0.98**	0.65	0.95	0.96
Ave.	**0.80**	0.77	0.67	0.77	0.60	**0.80**	0.71

The model incorporates innovative methods such as the measurement of attribute fluctuation based on membership degree variation and the construction of a fuzzy granular concept space to reduce redundancy. Additionally, it enhances concept representation by integrating similar fuzzy granular concepts, leading to the creation of a clustering space and the derivation of upper and lower approximation spaces. Experimental results on multiple real-world datasets demonstrate the superior performance of AFFCCL over traditional fuzzy similarity-based and CCL-based models in terms of classification accuracy, consistency, and robustness. Nevertheless, the model does have limitations: 1) sparse data can undermine the reliability of attribute fluctuation, and noise interferes with the authenticity of semantic association; 2) the model currently lacks a mechanism for dynamic updates, which limits its ability to handle real-time data. 3) AFFCCL excels on static datasets, but its architecture does not yet support real-time learning, large-scale, and streaming data. In the future, incremental learning frameworks, distributed computing optimization and concept drift processing will be developed to expand the application scenarios of the model. In future research, the focus will be on developing more lightweight CCL models to improve efficiency and scalability in high-dimensional data scenarios. This will involve optimizing computational complexity while maintaining classification performance. Additionally, enhancing the model's ability to handle incomplete fuzzy formal contexts and multi-view contexts, as well as, integrating real-time updates will be key areas of exploration. Expanding the application of AFFCCL to a broader range of domains and environments will also be an important direction for further development.

REFERENCES

[1] Y. Wang and Y. Wang. "Cognitive informatics models of the brain," *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, vol. 36, no. 2, pp. 203–207, 2006.

[2] Z. W. Pylyshyn. "Computation and cognition: Issues in the foundations of cognitive science", *Behavioral and Brain sciences*, vol. 3, no. 1, pp. 111–132, 1980.

[3] B. Ganter and R. Wille. "Formal concept analysis: mathematical foundations", *Springer Science & Business Media*, 2012.

[4] Z. Pawlak. "Rough sets", *International journal of computer & information sciences*, vol. 11, pp. 341–356, 1982.

[5] J. T. Yao, A. V. Vasilakos, and W. Pedrycz. "Granular computing: perspectives and challenges", *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 1977–1989, 2013.

[6] J. Wu, E. C. C. Tsang, W. Xu, C. Zhang, and L. Yang. "Correlation concept-cognitive learning model for multi-label classification", *Knowledge-Based Systems*, vol. 290, pp. 111566, 2024.

[7] H. -L. Yang, Y. -F. Zhou, J. -J. Li, and W. Ding. "A concept fringe-based concept-cognitive learning method in skill context", *Knowledge-Based Systems*, vol. 305, pp. 112618, 2024.

[8] W. Xu, D. Jiang, X. Zhang, and X. -Z. Wang. "A novel concept-cognitive learning model oriented to three-way concept for knowledge acquisition", *IEEE Transactions on Big Data*, pp. 1-14, 2025.

[9] J. Xu, E. C. C. Tsang, W. Xu, C. Zhang and S. Wang. "Multi-level correlation information fusion via three-way concept-cognitive learning for multi-label learning", *Information Fusion*, vol. 124, pp. 103361, 2025.

[10] L. A. Zadeh. "Fuzzy sets", *Information and Control*, 1965.

[11] C. Zhang, Z. Lu, Y. Zhang, and J. Dai. "Online Streaming Feature Selection Using Bidirectional Complementarity based on Fuzzy Gini Entropy", *IEEE Transactions on Fuzzy Systems*, 2025.

[12] J. Lu, G. Ma, and G. Zhang. "Fuzzy machine learning: A comprehensive framework and systematic review", *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 7, pp. 3861–3878, 2024.

[13] M. Sajid, A. K. Mali, M. Tanveer, and P.N. Suganthan. "Neuro-fuzzy random vector functional link neural network for classification and regression problems", *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 5, pp. 2738–2749, 2024.

[14] X. Deng, J. Li, Y. Qian, and J. Liu. "An Emerging Incremental Fuzzy Concept-Cognitive Learning Model Based on Granular Computing and Conceptual Knowledge Clustering", *IEEE Transactions on Emerging Topics in Computational Intelligence*, vol. 8, no. 3, pp. 2417–2432, 2024.

[15] J. Wang, W. Xu, W. Ding, and Y. Qian. "Multi-View Fuzzy Concept-Cognitive Learning With High-Order Information Fusion of Fuzzy Attributes", *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 12, pp. 6965-6978, 2024.

[16] Y. Ding, W. Xu, W. Ding, and Y. Qian. "IFCRL: interval-intent fuzzy concept re-cognition learning model", *IEEE Transactions on Fuzzy Systems*, vol. 32, no. 6, pp. 3581–3593, 2024.

[17] D. Guo, W. Xu, Y. Qian, and W. Ding. "M-FCCL: Memory-based concept-cognitive learning for dynamic fuzzy data classification and knowledge fusion", *Information Fusion*, vol. 100, pp. 101962, 2023.

[18] D. Guo, W. Xu, Y. Qian, and W. Ding. "Fuzzy-granular concept-cognitive learning via three-way decision: performance evaluation on dynamic knowledge discovery", *IEEE transactions on fuzzy systems*, vol. 32, no. 3, pp. 1409–1423, 2024.

[19] R. Wille. "Restructuring lattice theory: an approach based on hierarchies of concepts", in *Formal Concept Analysis: 7th International Conference, ICFCA 2009 Darmstadt, Germany, May 21 - 24, 2009 Proceedings 7*, pp. 314–339, 2009.

[20] M. -W. Shao, Y. Leung, X. -Z. Wang, and W. -Z. Wu. "Granular reducts of formal fuzzy contexts", *Knowledge-Based Systems*, vol. 114, pp. 156–166, 2016.

[21] J. Li, C. Mei, W. Xu, and Y. Qian. "Concept learning via granular computing: a cognitive viewpoint", *Information Sciences*, vol. 298, pp. 447–467, 2015.

[22] J. -H. Zhai, N. Li, and M. -Y. Zhai. "The condensed fuzzy k-nearest neighbor rule based on sample fuzzy entropy", in *2011 International Conference on Machine Learning and Cybernetics*, vol. 1, pp. 282–286, 2011.

[23] J. C. Bezdek, S. K. Chuah, and D. Leep. "Generalized k-nearest neighbor rules", *Fuzzy Sets and Systems*, vol. 18, no. 3, pp. 237–256, 1986.

[24] M. Sarkar. "Fuzzy-rough nearest neighbor algorithms in classification", *Fuzzy sets and systems*, vol. 158, no. 19, pp. 2134–2152, 2007.

[25] C. Zhang, E. C. C. Tsang, W. Xu, Y. Lin, and L. Yang. "Incremental concept-cognitive learning approach for concept classification oriented to weighted fuzzy concepts", *Knowledge-Based Systems*, vol. 260, pp. 110093, 2023.

[26] J. Niu, D. Chen, J. Li, and H. Wang. "Fuzzy rule-based classification method for incremental rule learning", *IEEE Transactions on Fuzzy Systems*, vol. 30, no. 9, pp. 3748–3761, 2022.